

Safe and Chaotic Compilation for Hidden Deterministic Hardware Aliasing

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Abstract-Hardware aliasing occurs when the same logical address can access different physical memory locations. This is a problem for software on some embedded systems and more generally when hardware becomes faulty in irretrievable locations, such as on a Mars Lander. We show how to work around the hardware problem with software logic, compiling code so it works on any platform with hardware aliasing with hidden determinism. That is: (i) a copy of an address accesses the same location, and (ii) repeating an address calculation exactly will repeat the same access again. Stuck bits can mean that even adding zero to an address can make a difference in that environment so nothing but a systematic approach has a chance of working. The technique is extended to generate aliasing as well as compensate for it, in so-called chaotic compilation, and a sketch proof is included to show it may produce object code that is secure against discovery of the programmer's intention. A prototype compiler implementing the technology covers all of ANSI C except longjmp/setjmp.

I. INTRODUCTION

Hardware aliasing describes "the situation where, due to either a hardware design choice or a hardware failure, one or more of the available address bits is not used in the memory selection process." [1] The effect used to be familiar to programmers and users alike, as the 'DLL hell' that the old 16-bit versions of Windows were prone to. Dynamic linked libraries (DLLs) were problematic for many reasons, but one was that different versions of the same library loaded at the same memory address and all applications referenced the in-memory copy. So if one application loaded one version of the library then another loaded another version, the first application would unpredictably, as far as the program and user were concerned, find itself in the second library's code.

DOS users truly experienced the situation in the raw, as the ubiquitous expanded memory managers (for memory beyond 1MB) such as QuarterDeck's QEMM [2] remapped memory so the video graphics (VGA) and bootstrap (BIOS) code shared addresses with random access memory (RAM). What a program accessed at runtime at a given address depended on the memory manager heuristics. In consequence, code had to use standard sequences to trigger expanded memory reliably. Nowadays, processors such as the Raspberry PI-2 (all models) [3] still dynamically share up to 1GB of RAM for general-purpose use with the graphics processing unit (GPU).

Whatever the physical rationale, the symptom is the same in every case. It is that what looks to programs to be the same memory address (the *logical* address) sporadically accesses physically different resources. Since the memory management hardware in modern processors maps all input/output (I/O) into one monolithic processor address space, that may mean different regions of RAM or it may mean different peripheral devices such as USB ('universal serial bus') and GPU.

Certainly, modern applications programmers are more familiar with software aliasing, in which the same physical resource - memory location or peripheral device - is accessible via different logical addresses. But hardware aliasing has not gone away so much as become less relevant for the ordinary programmer as platforms have evolved to present a normalised view to software. The paradigm of platform hardware that actually has hardware aliasing through and through but presents 'normally' to even a systems programmer is oblivious RAM (ORAM) [4], [5], [6]. That is a secure RAM solution (available in various forms since the 90s) in which internally the memory contents are continuously randomly aliased and re-aliased to frustrate cold boot attacks [7] (freezing the RAM sticks to make the electrical charge last longer for analysis; RAM chips require a read-and-rewrite cycle to maintain their contents and ordinarily taking them out of the processor would lose the contents immediately). But the closer the programmer works to hardware the less complete may be the facade. On platforms which are both resource-limited and relatively inaccessible, programmers have to work around not only hardware that shares the same address space but faults that can exacerbate the situation. A stuck bit may take one address line out of commission, forcing the hardware map into a smaller shared address space. Or a Mars Lander may suffer a cosmic ray through the processor that causes the arithmetic logic circuit to compute 1+1=3. One way to cope with that is to rewrite programs to treat 2 and 3 as indistinguishable for the purposes of arithmetic. That means rewriting to work in arithmetic modulo a suitable equivalence. That may be, for example here, a partition of the number space into pairs $\{2,3\}, \{4,5\},$ etc., in which $\{2m, 2m+1\} + \{2n, 2n+1\} = \{2(m+n), 2(m+n)+1\}$ and $\{2m, 2m+1\} * \{2n, 2n+1\} = \{2mn, 2mn+1\}$. That equivalence should in general be the least indiscriminatory that makes the processor's faulty answers x+y into a homomorphic image of standard computer arithmetic.¹ That entails losing one arithmetic bit or more, since the only homomorphic images have sizes that are factors of the original size 2^n , which will be 2^m for some m < n.

¹The simplest construction of a suitable equivalence is to set the processor's wrong result 1+1 = 3 equivalent to the correct result 1+1 = 2, with $2 \equiv 3$, and then close under the implications $x_1 \equiv x_2 \land y_1 \equiv y_2 \rightarrow x_1 + y_1 \equiv x_2 + y_2$, where the + is the correct one. Similarly for multiplication, etc. The equivalence shown is $x \equiv y$ iff $\lfloor x/2 \rfloor = \lfloor y/2 \rfloor$, with $2 \equiv 3$.

While 4 and 5 may denote the same value arithmetically, as addresses they access different memory locations and a program rewritten to accommodate a nontrivial arithmetic equivalence will experience hardware aliasing. So the cure for the computational mathematics engenders a need to handle the consequential aliasing. Since low-level programs in particular use pointers heavily, there is no room for an ad hoc approach to that. Even high-level programming languages use pointers nearly ubiquitously - every object in Java is really a pointer, so copying an object requires the clone() system call. A simple copy will just be another pointer to the same object. Every variable in Fortran95 is a pointer – a subroutine call f(x) alters the value of its caller's variable x when the subroutine writes to x. Arrays in C are passed as pointers too. Not only will system-level programmers need a reliable way to program around hardware aliasing because systems programming is hard enough without new traps for the unwary, but applicationlevel programmers, the mathematicians and control engineers, must be able to remain comfortably ignorant of the hazard.

It turns out that there is a systematic way of generating code that always works in a hardware aliasing environment - a compiler mechanism - and this paper sets it out. There has been no other work on this topic since the present authors made an original suggestion as to what to do in 2014 [8], but experience has shown that suggestion to be too delicate for practice (the reference is anonymised but the idea was to set up different type classes for arrays – in one, array entries are uniquely accessed by walking a pointer in unit increments up from the bottom of the array, in another, access is uniquely by walking down from the top, in another, access is uniquely via a single constant offset from the bottom of the array, and so on; each class enforces a unique calculation for each array entry which produces the same result each time, as discussed below, but the programmer must choose what type to declare each array as and there is no proof that deeply nested arrays of pointers to arrays of pointers, etc., will not give rise to inconsistencies). The technology reported in this paper is less efficient computationally but it is robust and usable in practice by programmers, and it has been deployed and tested - c.f. the HAVOC compiler suite for ANSI C [9] at http://sf.net/p/obfusc. The programmer is aware only that each pointer must be declared with restrict. That modifier nominates a top-level (or local) memory zone that the pointer is to range through. Not declaring a zone means the compiler assumes the pointer can point anywhere at runtime, which causes prohibitively large object code to be generated - gigabytes for a line of source code – so it behoves the programmer to declare a small zone.

The solution works because of an underlying determinism in processors. Processors are mechanisms that are both designed to produce repeatable results and also do so inherently because of their material. They are quite high power electronic logic circuits, which makes it rare for a faulty bit to be really random. Even if the electrical value is floating then that will give one determined value when tested electrically in a determined way – in a semiconductor circuit the test is always either if the bit can supply current to ground or to the positive rail and the answer for a floating line is 'no' both ways. The logic circuit has been damaged so that it is different from the way it was designed but it is still a logic circuit which responds to given inputs with given outputs, in this hypothetical case producing a floating bit in some conditions. The functionality depends on the circuit around the damaged area and it may be more complicated input to output than always on or always off, but it should be deterministic.

The situation may be encapsulated as *hidden deterministic* hardware aliasing, axiomatised as follows with respect to the addresses produced by sequences of processor operations: Axioms

- 1. A machine code copy instruction copies the physical bit sequence exactly, such that a copied address accesses the same memory location as the original;
- 2. repeating the same sequence of operations produces an address that physically has exactly the same bit sequence and accesses the same location;
- 3. logically different addresses always have physically different bit sequences.

By 'logically different' is meant different in terms of the intended arithmetic. In the example, 4 and 5 are not logically different as they are equivalent in the equivalence relation with respect to which arithmetic is well-defined, with $4 \equiv 5$, but 5 and 6 are logically different and $5 \neq 6$ in the equivalence relation. The 4 and 5 can be thought of as different physical encodings of the same logical number (here, 2), while 5 and 6 are different physical encodings of different logical numbers.

Axiom 1 ('faithful copy') implies a compiler can generate code that copies an address for later use after writing through it, and the copy can be trusted to retrieve the written value. The address must not be altered, not even by adding zero, as any arithmetic calculation at all potentially alters the physical representation of the address as a sequence of bits, which then fails to access the same memory location. In the example given, 0 is equivalent to 1, so adding what is arithmetically zero may in fact mean adding one in physical terms.

Axiom 2 ('repeatability') allows for some calculations on addresses, so long as they are repeated exactly each time. That is useful because machine code instructions to read or write a memory location generally take a base address a in a register and adds a displacement d embedded in the instruction to get *effective address* a+d for the access. It is impossible to avoid the processor doing that one addition, but it does not matter because the same calculation is repeated at each access, with physically the same sequence of bits resulting.

Axiom 3 ('no confusion') guarantees that the representations as physical sequences of bits of what are logically different addresses do not step on each other. The addresses with bit sequences 5 and 6 in the example represent logically different numbers (logically 2 and 3 respectively).

The layout of this paper is as follows. Section II explains how to compile to obtain code 'safe against hardware-aliasing' in the most general context. The generated code has to cope with variations in the addresses produced, while they are actually governed by a 'hidden determinism', as described above. Section III details an extension in which variations are deliberately introduced by the compiler (which can cope with them) 'to the maximal extent possible' as described in in the section, and that is called *chaotic compilation*. The section finishes with an argument that unrolling code to a length that is exponential (super-polynomial) in the number of bits per word and then compiling it chaotically results in safety against polynomial complexity attacks aimed at determining what the data in the runtime trace means. The technique introduced in Section II can be seen as coping with apparently unreliable *addressing* (really deterministically generated) and the technique of Section III extends that to deterministically generate apparent unreliability in *data* in general, and cope.

II. SAFE COMPILATION FOR HARDWARE ALIASING

The working principle for generating viable code in this kind of environment is that each address that is written should be saved for later read, as per Axiom 1 ('faithful copy') of Section I. The problem is that it is saved at an address, which must also be saved, and so on recursively. Axiom 2 ('repeatability') allows a backstop to be put on the recursion, via base addresses that are produced at compile time and a repeated calculation at runtime based on them. But no finite set of addresses can suffice for nested function calls to unbounded depth, so the runtime stack must be involved. The first problem is how to manipulate the *stack pointer* so addresses and other data might be saved and recovered reliably from stack.

A. Stack Pointer 101

The standard compiler-generated function call sequence decrements the stack pointer register **sp** by the amount that will be needed for local storage in the function immediately on entry to the function body, and increments it again before exit:

That does not work in a hardware aliasing environment, because the increment does not necessarily restore exactly the same physical representation originally in the stack pointer register. Instead, the caller gets back a possibly different set of bits that, however, means the same thing arithmetically. Being different, it references a different location in memory.

The *frame pointer* register **fp** must be co-opted to copy the stack pointer, and copy the stack pointer back from it just before return from the call:

That is the standard unoptimised function call sequence from a

compiler but optimisation will replace it with the **fp**-less code (1). The GNU *gcc* compiler (for example) with **-fno-omit-frame-pointer** on the command line turns off optimisation and produces the code (2), which works with hardware aliasing.

It is not perfect, however, because the caller's frame pointer register must also be saved and restored by the callee around its own use of the frame pointer register, as follows:

function code start:	
save old fp to 1 below sp	
copy sp to fp	
decrement sp	(3)
	(5)
copy fp to sp	
restore old fp from 1 below sp	
return	

Saving below the caller's stack pointer would ordinarily intrude on the callee's stack area ('frame'), so the decrement must be larger by one in order to leave room for it. As many as the compiler wants of the caller's registers can be saved like this. The application binary interface (ABI) document for the platform specifies which registers the callee must save, and which the caller code must expect may be trampled on and must save itself. The frame pointer and stack pointer are callee-saved in (3).

The function call code (3) works well with hardware aliasing. It allows local variables for the function to be reliably addressed as sp+d on the stack, where d is a displacement between 0 and the function frame size. The d is supplied as an embedded constant in a *load* or *store* machine code instruction (see below in (4-5)) that references the stack pointer register, containing sp, as the base for the displacement. The processor calculates sp+d for the *effective address* passed to memory.

B. Variables

With the function call sequence in (3), accessing local variables is simple. A word-sized local variable \mathbf{x} is assigned a position n on the stack and the compiler issues a load instruction (**lw**) to read from there to register r:

lw r n (sp) # load from offset n from sp (4)

The processor calculates sp+d but repeats the same calculation at every access, so by Axiom 2 of Section I the same sequence of bits for the address is produced every time, and it accesses the same spot in memory. To write the variable, a *store* instruction (**sw**) replaces the load instruction:

$$sw n(sp) r \# store to offset n from sp$$
 (5)

For a global variable at an address a in (heap) memory, the compiler offsets from the zero register **zer** instead:

$$lw r a (zer) \# load from address a$$
 (6)

The zero register contains a fixed base value. The effective address sent to memory is a+0, which is possibly a physically different sequence of bits to a (representing the same value), but the calculation is repeated exactly each time so the same memory location is accessed each time.

Variables in the parent's frame may also be accessed. If the function is defined within another function, we shall call it an *interior* function ('nested' is the standard term, but it risks confusion with 'nested' function calls – one function called from another). The compiler arranges that the exterior function's frame pointer is handed down at runtime in the **c9** register (it is **c9** in our own platform's ABI; it may be different in other ABIs) and that is preserved through successive interior function calls. Then a load or store instruction using **c9** instead of **sp** or **zer** reliably accesses the exterior function's variables on the stack.

C. Arrays

Arrays present the real difficulties as entries are fundamentally always accessed by address and addressing is fundamentally unreliable in a hardware aliasing environment.

There are (at least) two common but different ways of addressing the entries in an array a. That is (a) via a load or store instruction with fixed displacement n from the array address a (that is 'a[n]'), or (b) via a pointer with value p that ranges through the array starting at a and steps through the elements until the desired one is reached, at which point a load or store instruction with displacement 0 from the pointer (p[0]) is applied. Those two modes were considered in our earlier work [8] (an anonymised version will be available from the Chair), which continues to be the only work we know of on how to even possibly get around hardware aliasing. The two calculations (a) and (b) for the effective address are respectively a + n and $a + 1 + 1 \cdots + 1 + 0$. The calculations may produce physically different sequences of bits to be sent to memory, so the two are mutually incompatible and one or the other must be used all the time for consistency. But both have proven too restrictive in practice. It is as common, for example, for real code to step a pointer down through an array as to step up through it, and the transformation the compiler needs to do results in prohibitively inefficient code at runtime. Evidently also, there are many other ways of addressing array entries commonly encountered in source code and we cannot be expected to cater for them all.

We have accepted instead an engineering compromise in which array access is not for general-purpose use going to be constant time. For special purposes, one of the addressing modes (a) and (b) may be used, but that would be liable to cause programming mistakes at application-level. For arrays of size N it turns out the compiler can provide access in $\log N$ time in a simple manner that is safe and reliable for all methods of calculating an index or pointer, known or unknown.

Linear complexity code will be presented first. To read (local) array element $\mathbf{a}[\mathbf{n}]$ the code tests \mathbf{n} against each of $0, \ldots, N-1$ in turn and chooses one address for each entry:

$$(n == 0)?a[0]: (n == 1)?a[1]:$$
 (7)

The equality tests are arithmetic and are therefore insensitive to the physical representation of the value n of **n** as possibly physically different sequences of bits. The generated machine code always passes effective address a + d to memory where d is the displacement from the base of the array for the entry and a is the address sp + k, where k is the position on the stack allocated by the compiler for the lowest array element $\mathbf{a}[\mathbf{0}]$. The code is just the one machine instruction:

$$l \le r \ d \ (r) \qquad \qquad \# \ \text{load from address} \ a+d \\ \# \ \text{with} \ a \ \text{in} \ r \ \text{to} \ r$$
 (8)

The address a is supplied by a preceding instruction:

addi
$$r$$
 sp k # add k to sp in r
 $\lim r d(r)$ # load from address $a+d$ (9)

That produces the same calculation sp+k for a every time.

Improving this code to $\log N$ complexity means using a binary tree instead of linear lookup for the value n of **n**, deciding first if n is below N/2 or above it, then on what side of N/4 or 3N/4 it is, and so on. Code for writing instead of reading follows the same pattern, with store instead of load instructions at the leaves of the binary tree or linear sequence.

The same form works for pointer \mathbf{p} accesses, provided the compiler knows what zone of memory it points into. We have tightened the type system of C (the source language for our prototype compiler) so the pointer is declared with the name of a (possibly over-large) array \mathbf{a} into which the programmer guarantees it points at runtime:

That selects array **a** as the target zone for **p**.

Some porting has to be done for existing code, marking out areas into which different pointers point. It generally turns out to mean declaring a global array from which objects of the kind pointed to are allocated from, or declaring one function as interior to another function (for example, main) where the target zone of the pointer is declared as local on the stack.

A **restrict** pointer type is narrower than it would otherwise be but there are no no semantic changes to the language so the programmer does not have to relearn anything. The programmer does (usually) have to make changes, but the new code is valid in unmodified C, so it can be checked. Ideally it is obtained via a sequence of careful code transformations. The conservative choice is usually to replace ad-hoc declarations with calls to an object factory for each compound type.

The following code is suitable for lookup via pointer **p**:

$$(p == a+0)?a[0]:$$

(p == a+1)?a[1]: (11)

It is insensitive to the way the pointer \mathbf{p} is calculated because the equality tests are arithmetic and are not derailed by the aliasing in memory addressing. A selection $\mathbf{a}[\mathbf{d}]$ is made, and the same lookup code (8) for $\mathbf{a}[\mathbf{d}]$ is executed each time, giving the same result.

This code can (also) be made $\log N$ complexity with a binary tree structure. It can be converted for write by replacing load instructions at the leaves with store instructions.

These constructions make pointer-based addressing consistent with access via an array index in the hardware aliasing context. The idea is to make the choice of address arithmetically, and then reuse that same address or calculation for the address again and again.

If the number N of array elements is determined at runtime (the C '99 standard adds so-called *variable length arrays*), then a 'late binding' code pattern is needed instead. The following generated code generalises (11) to read reliably through pointer \mathbf{p} for a number N of elements that is a local variable at runtime:

The effective address passed to memory is $sp + k + (0 + 1 + \cdots + 1) + 0$, where a = sp + k is the address of array **a**, and the 1s are repeated d times to address the dth element of the array. This is not the same calculation as in (11) so it is not generally compatible with that in this kind of environment. The compiler must always generate either the form (11) or the form (12). If there are no dynamically sized arrays in the code then it can afford to use (11) (the generated code is longer but conceptually simpler). If there are dynamically sized arrays then it must use the form (12). Tighter distinctions may be possible, but we have not explored them.

The same form must be used for indexed read:

The calculation for the effective address passed to memory is the same as in (12). Both (12-13) can be modified for $\log N$ complexity, and then the modified form must be used always.

Writes replace the 'a[d]' return (translated to a load machine code instruction by the compiler) with a 'a[d] = x' (translated to a store machine code instruction).

D. Data Types

We will not unduly belabour the topic here, but compound (and short) data types need special consideration, at least because compound data may contain arrays and arrays need special consideration (above). Indeed, a common programming style in C is to access the members of a compound data type as though the whole were an array of words, and to access the members via a pointer to word. Unless debugging memory pointer problems is a favourite pastime, the programmer wants the compiler to get both modes of access (conventional and as an array of words) right.

The trick for the compiler is to always treat records with named fields ('struct' in C) as word arrays and translate the field name to an array displacement. The declaration

struct { int a; int b; } x
$$(14)$$

declares **x** with two named fields, **a** and **b**, each one word wide. It occupies two words on the stack at displacements kand k' (the value k+1) respectively from the stack pointer. The compiler generates accesses to the fields **x.a** and **x.b** just as it would for any local variables situated there, by calling

$$lw r k(sp) \# load from x.a$$
(15)

to read from **x.a**, for example. The effective address passed to memory is sp+k. To access **x.b**, the address passed is sp+k' instead. The compiler generates the code for array access explained in the section above, and source code that accesses the fields of the struct as though it were an array works too.

Long atomic types such as **double** are also treated as arrays of words by the compiler. But most platforms have doubleword load and store instructions that will fetch/write two words at once:

$$ld r k (sp) # double word load (16)$$

and only the address of the first word is given to the instruction. Registers are indexed as pairs for this instruction, and the partner to r is loaded up by the instruction with the word at address k+1.

But that is not necessarily compatible with treating the double as a two-word array in a hardware aliasing environment. If the double is on the stack then the effective address for the first word is sp+k, say. The processor may request a double-word from memory at that address. Or it may request two words, one at sp+k and one at (sp+k)+1. Both address calculations are reliably repeatable, according to the axioms of Section I, but by those axioms there is no guarantee that the latter calculation accesses the word at one beyond sp+k in memory. Indeed, the example given in that section shows that if sp+k gives one of the answers $\{4, 5\}$, then the arithmetic must be so arranged that (sp+k)+1 gives one of the answers $\{6, 7\}$. It may be that 4 is produced for the former, and 6 for the latter, and those are not consecutive positions in memory.

Overall, it is safer that the compiler *not* make use of double word instructions, though if the program contains no accesses to doubles as two-word arrays, then it is perfectly safe.

The difficulty remarked above with **double** transfers to accessing the individual bytes of a single word too. It is safer overall not to use the byte-oriented instructions that the platform provides, but to use arithmetic instead, as follows.

For index-oriented access to the characters of a string \mathbf{a} , the compiler generates code that splits the character index \mathbf{i} into index \mathbf{d} for a word consisting of a sequence of 4 characters, and offset \mathbf{j} for the wanted character within the word:

$$d = i/4;$$

 $j = i$ %4; (17)

Then the character is obtained via an array-of-words lookup and the following arithmetic for the jth char of the dth word:

In our own prototype compiler, we have preferred to avoid the complication and pack characters only one to a word, at the cost of an inefficient use of memory.

III. DELIBERATELY CHAOTIC COMPILATION

We can take the general scheme for compiling around hardware aliasing on the platform and use it to work around variations introduced by the compiler itself. The point is that varying the object code tends to obscure what it does, and obfuscation is a legitimate aim of some fields of software engineering. Many companies might like to distribute object code that is so obfuscated that it cannot be reverse engineered, while not being hampered in execution (of course it is easy to produce code that theoretically cannot be reverse engineered – it suffices to include a loop that in parallel to everything else that the program does, searches at low priority for an inconsistency proof for integer arithmetic, and if it finds one, interrupts the rest of the program, and nobody can say for certain if the program will complete normally or not – but in practice a skilled engineer can make a good job of it).

The aim in chaotic compilation is to compile the same source code differently each time, forming object codes that have the same overall functionality but the runtime traces vary to the maximal extent possible within these constraints:

- 1) The same sequence of machine code instructions executes, in the same order.
- The constants embedded in each machine code instruction differ to the maximal extent possible.
- 3) The data written to registers and memory differs to the maximal extent possible.

The idea is to compile program p_1 and prepare data d_1 for it, and to compile program p_2 and prepare data d_2 for it. The two data sets represent the same thing, but their presentation for the two compiled programs differs. Then $p_1(d_1)$ and $p_2(d_2)$ are run. Comparing the two runtime traces, the characteristics 1)-3) above are seen. The 'differs to the maximal extent possible' refers to the stochastic distribution of outcomes. Compilation is randomised, so each runtime trace is one of a range of possible traces, and the ideal is that each trace is exactly as frequent an outcome as any other among the possibilities. That is a 'flat' distribution of outcomes, which is a maximal entropy² distribution in information theoretic terms. That is, no bias or other tendency is discernible. Even though human programmers naturally tend to use low numbers like 0 and 1, the compiler will have randomised the object code so that tendency, and any other, is not present. So 'differs to the maximal extent possible' means a flat probability distribution of the stated observations, which is technically *chaotic*.

'Getting the intended result' would be a fourth constraint, but it is already said that the overall functionality is retained.

It will be argued that chaotic compilation may be applied to any program such that on a platform with a n-bit word:

The runtime trace cannot be read correctly with probability above chance by a polynomial time method (\mathcal{O})

in n as $n \to \infty$, in the (hypothetical) situation that the hardware word size can be varied. That is, the probability p of getting right what it is intended by the programmer for any chosen data bit in the trace to mean tends to 1/2 as $n \to \infty$.

In the simple case of a program with no code, for example, the compiler produces no machine code for both programs p_1 and p_2 above. The prepared data d_1 and d_2 still differ, however. For example, in d_1 the author may write 3 where they really mean 7, but in d_2 they may write 15 where they really mean 7, and so on. The author can still read the output (which is the same as the input) because they know the substitution. But an onlooker cannot tell what they really meant. There is no bias in the observations if the presentation scheme for the input data (which is the same as the output) is randomly chosen (this is a fundamental result of information theory – the entropy in two *n*-bit signals added together cannot be less than the entropy of either, so when one has maximal entropy, the combined signal also has maximal entropy, which means a flat distribution).

The more remarkable part is that the reasoning works for any program. We will show how the compiler may vary

- A. addresses (Subsection III-A below);
- B. data content (Subsection III-B)

randomly so traces 'differ to the maximal extent possible' at runtime between compilation and recompilation, elaborating the technique of Section II to induce variation as well as cope with it, concluding with a sketch of the argument for (\mathcal{O}) .

A. Address Displacement Constants

Instead of generating a load instruction to read from a variable at position n on the stack like this (8):

 $l \le r n(\mathbf{sp})$ # load r from offset n from sp

the compiler will issue the instruction with a *displacement* constant Δ different from n:

 $\lim r \Delta(s) = \#$ load from offset *n* from *sp* (19)

 Δ is randomly chosen and the register s has been pre-set to contain $sp + n - \Delta$, where sp is the nominal value of the stack pointer. The bit sequence passed to memory by (19) as effective address to read from is the result of the calculation:

$$sp + n - \Delta + \Delta$$
 (20)

The compiler always emits the same instruction sequence, getting the same address always as result, but it has to ensure that the Δ used in place of the *n* is the same each time, for each *n*. So it maintains a vector Δ indexed by stack location *n*. (a similar vector Δ_Z is maintained for the heap). Then the Δ in (19) is really $\Delta = \Delta n$ and the instruction in (19) is:

$$\lim r \Delta n(s) \# \text{ load from offset } n \text{ from } sp(21)$$

The vector Δ is changed randomly by the compiler as it works through the source code and is a source of extra variation, but as explained above it seems not to do anything because the base address read from register s is asserted in (20) to be offset to compensate. That is not true. The compiler controls the offset in register s too, as explained below, so that random variations from nominal in every register and memory location are produced and accounted for. The essence is that:

Each machine code instruction that writes is freely varied to the maximum extent possible. (\bar{h})

For example, the load instruction of (21) has one embedded constant Δn and it is freely varied by the compiler. The

²The entropy of a distribution of random events X is formally the expectation $E[-\log_2 \operatorname{prob}(X)]$. Informally, it captures the number of 1/0 degrees of freedom ('bits'). 1 bit equates to two equally probable possibilities.

instruction is exceptional, however, in that it writes, but not in any way that is controlled by the varied constant. The result always ends up in the named register r. Instead 'writes' must be understood as 'having an observable effect on the trace'.

B. Content Deltas

As remarked above, the stack pointer **sp** does not contain the value *sp* that it notionally should have but instead is offset from that by a randomly generated value δ . That is true of the content of every register and memory location at every point in the generated code. The compiler maintains a vector \eth of the offset delta for content in each register and memory location, varying it as it passes through the code, and the δ for the stack pointer register is $\delta = \eth \mathbf{sp}$.

We need to work through enough detail of what the compiler does to show the principle (\overline{h}) is satisfied, but most will be omitted. The following is an abstract, declarative rendering of what the compiler does to translate a non-side-effecting expression *e* of the source language, which is the simplest part of its work.

Let the compiler be $C_r[-]$, translating *e* to machine code *mc* that targets the result for register *r* at runtime. That is:

$$(mc, \eth) = C_r[e] \tag{22}$$

Let the state of the runtime machine before mc runs be σ_0 , let the nominal value for the expression³ be $[e]^{\sigma_0}$, then running code mc takes state σ_0 to state σ_1 in which the value in register r is offset from the nominal value by the randomly generated amount $\eth r$. That is:

$$\sigma_0 \stackrel{mc}{\leadsto} \sigma_1 \quad \text{where } \sigma_1(r) = [e]^{\sigma_0} + \eth r$$
 (23)

Re-rendering the code in (21) to read the *n*th location on the stack and show explicitly the previous instruction that preps the base address (in register r) for the load:

addi r sp
$$k$$
 # $k = n - \eth sp - \Delta n$
lw $r \Delta n(r)$ # read nth stack location (24)

The memory receives as effective address to read from the result of the following calculation:

$$sp + \eth \mathbf{sp} + k + \mathbf{\Delta}n$$
 where $k = n - \eth \mathbf{sp} - \mathbf{\Delta}n$ (25)

The stack pointer register **sp** contains the nominal value sp offset by \eth **sp**. Summing, the address has the arithmetic value sp+n. The physical representation as a sequence of bits may both be different from nominal and may vary, as discussed in Section I: one might equally get a 4 as a 5 from the calculation, following the example there. However, the calculation is the same every time so the bit sequence passed to memory is the same every time, by Axiom 2. It cannot hit the memory location associated with any other (logical) address by Axiom 3, and the calculation for the address (25)

is repeated exactly at every access via (24) so it hits just one memory location of those feasible for the logical address.

The \eth and Δ values are changed by the compiler at (just before) every point in the code where a write occurs. After the write the deltas for that location have to be maintained constant through the following sequences of reads from the same location along the code paths through that point, or the reads would miss. There are two instructions generated in (24) and both carry constants that can be varied by the compiler. However, the constraint on the right in (24) binds them and restricts the total variation possible. A further variation beyond that constraint would to vary the logical position n of the target on the stack at every write to it. That can be done but it is too tricky to describe here, so we will hypothesise that the position on the stack forms part of the intended program semantics and it cannot be varied.

Within that constraint, the two instructions in (24) can be varied maximally by the compiler, as required by (\overline{h}). It is just that, by virtue of the constraint, the entropy the compiler can introduce to the runtime trace via them is not 2×32 bits (assuming a 32-bit word), but only 1×32 bits. That is, if the Δn constant in the load instruction is freely chosen by the compiler, then the k constant in the preceding addition is determined by $k = n - \eth \operatorname{sp} - \Delta n$, and if the k constant is freely chosen then the Δn value is determined.

Closely related work presently under review [10] makes the above observations precise:

Theorem 1. The entropy in a trace over recompilations is the sum of the entropies of every instruction that writes that appears in it, counted once each.

The entropy of the two instructions combined in (24) is 32 bits (on a 32-bit platform). That can be seen as 32 bits for the first instruction, and zero for the second, as its variation is already determined once the first has been seen.

The compiler's job in the chaotic compilation context is to do everything it can to maximise entropy in the trace. It turns out that following (\overline{h}) as a compiler design principle does that, which is why we have been careful to check for it above:

Theorem 2. The trace entropy is maximised when the compiler varies every instruction that writes to the maximal extent possible from recompilation to recompilation.

Successfully varying each instruction to the maximal extent possible provides a stochastic setting at runtime in which an observer cannot be sure what the numerical value of the data in the trace is really intended to mean even in terms of a statistical tendency, because the maximum 32 bits (on a 32-bit platform) of information caused to be written by the programmer at each point in the trace is swamped by that much contribution again from the compiler. But there are limits on what the compiler can do in terms of variation, as argued for (24) above.

But the constraint for (24) can be removed via a modification to the underlying platform. The idea is to allow potentially any address to stand for 'the *n*th location on the stack'. That requires an extra hardware or software address translation unit

³The 'nominal value' $[e]^{\sigma}$ of expression e is is formalisable via a canonical construction: map a variable x in the expression to its register location r_x (the runtime value is offset by a delta ∂r_x), checking the content of r_x in the state and discounting the delta to get $[x]^{\sigma} = \sigma(r_x) - \partial r_x$. Arithmetic in the expression is formalised recursively, with $[e_1 + e_2]^{\sigma} = [e_1]^{\sigma} + [e_2]^{\sigma}$, etc.

between the runtime software process and memory. On being passed a new address a intended as the *n*th stack position, it remaps it to the next free address b in a previously designated contiguous linear region of memory R, say the region between 1GB and 1.25GB, and memoizes the choice so the next time the address a is passed to it, it is mapped to b in R again. That gives each process access to 250MB, though the addresses ait generates range randomly across the full 32-bit range (0 to 4GB). The compiler inserts instructions in the object code that remove defunct mappings, keeping the memory needed down.

That unit exists and is part of every processor. It is the 'translation lookaside buffer' (TLB), and its job is to remap memory address space a page (8KB) at a time, but we need it to work with individual addresses, not whole pages. An easy solution is to simulate it in an underlying software layer. Given that, the compiler is free to vary n in (24) and the constraint in (25) relates three constants, k, n and Δn , allowing for 2×32 bits of entropy for the two 'instructions that write' in (24), satisfying (\overline{h}) and allowing Theorem 2 to conclude the trace has been randomised ('trace entropy is maximised').

But there are still other constraints on compiler-induced variation due to computational semantics. The inputs and outputs of a copy instruction are the same, so the variations in them are the same and not independent. Also, the compiler must set the same variations from nominal values at the end of a loop as at the beginning, because it cannot tell in general how many times a loop will be traversed at runtime and must prepare the code for another traversal after one time through.

We will not go through all the code constructs, just loops, to show how and when the principle (\overline{h}) must sometimes fail.

C. Loops

Let the statement compiler C[-] produce code mc from statement s of the source language, changing the combined database $D = (\Delta, \eth)$ of offsets to D^s in the process. The offsets \eth are the intended variations from nominal values at runtime for the data content of each register and memory location and the offsets Δ are the extra displacements for addressing described in Section III-A.

For legibility, pairs (D, x) will be written D : x here. Then the statement above of what the compiler does with statements s is formalised as:

$$D^s: mc = C[D:s] \tag{26}$$

The notation emphasises the compiler is side-effecting on D.

Compiling while es means emitting code mc constructed from mc_e for e and mc_s for s, with this shape:

start: mc _e	# compute e in r	
beqz r end	# goto to end if r zero	
mc_s	# compute s	(27)
b start	# goto start	
end:	-	

That does not work as-is, because the code does not at the end of the loop reestablish the deltas that prevailed at loop start, so a second time through at runtime, much goes wrong. Extra code is needed after mc_s , so-called 'trailer' instructions. A trailer instruction adjusts the content of register r back to the delta ∂r off nominal at the beginning of the loop, starting from a delta $\partial^s r$ at the end of the loop. It is as follows:

addi
$$rrk$$
 # where $k = \eth r - \eth^s r$ (28)

Just the one instruction is required per register.

Trailer instructions that restore the nth stack location offset must also restore the address displacement constant used in load and store. That is a more complex code sequence:

addi **t0 sp**
$$j$$
 # $j = n - \eth$ **sp** $- \Delta^{s} n$
lw **t0** $\Delta^{s} n$ (**t0**) # load *n*th stack location
addi **t0 t0** k # modify by $k = \eth n - \eth^{s} n$ (29)
addi **t1 sp** l # $l = n - \eth$ **sp** $- \Delta n$
sw Δn (**t1**) **t0** # store *n*th stack location

(the t0, t1 registers are 'temporary' workspace). The first two instructions are the read stack code of (24), and the last two are the corresponding write code. In-between, the instruction (28) changes the content delta. The sequence reads with one address displacement, and writes with another, changing the location of the (changed) content.

The trailer sequences (28-29) introduce no entropy at all into the runtime trace and therefore fail the principle (\overline{h}) (effectively, per instruction that writes, 32 bits of variation is needed from the compiler to mask programmer information). The constants in the instructions are determined by choices of deltas by the compiler for earlier instructions and it is impossible to execute the trailer instructions without traversing the loop body, which will execute those earlier instructions, so these trailer instructions are always 'old news' when they run. The content of the registers or memory locations they affect during execution is correlated with the content of registers and memory locations at the start of the loop.

Other places where the compiler must put trailer instructions like (28-29) are where conditional branches join again, the labelled targets of **gotos**, and at **return** from functions. Calls of interior functions also require 'trailers', but before the call, because the delta offsets in force at the point where the function was defined must be reestablished (that is a 'come from' semantics; c.f. **goto**). If there are global variables, then the same goes for ordinary (i.e., not 'interior' but top-level) function calls with respect to those globals they access.

At each of those points the principle (\overline{h}) fails, because every instruction that writes (on a 32-bit platform) does not introduce 32 bits of entropy from the compiler.

D. The Argument for (\mathcal{O})

Given the analysis above, the argument for (\mathcal{O}) requires a program to be unrolled far enough before chaotic compilation so none of the points where (\overline{h}) fails will be encountered in practice by an observer. Then by Theorem 1 the entropy in the observed part of the trace is sufficient to mask completely any input from the programmer, and an observer cannot say with any significant probability of being right (as the word length *n* tends to infinity) what any bit of data picked out of the trace means. A fuller sketch of the argument is as follows: **Proof:** [Sketch for (\mathcal{O})] Suppose for contradiction that the observer has a polynomial time method f(n, T) of working out what the data beneath the encryption is at some m points of interest in the trace T of program P, where n is the platform word size. For brevity assume P includes its intended inputs, compiled-in. The observer sees runtime trace T.

WLOG take m=1 and suppose the observer is interested in only one particular bit in that particular register or location in memory at that point, say the least significant bit b. It is last written to in the Mth step of the trace before the point of interest. Then the observer's prediction f(n,T) is 0 or 1. Suppose also the program has been written in the machine code subset consisting of addition instructions with semantics $x \leftarrow y+k$ (addi x y k) for a constant k embedded in the instruction, and branch instructions with semantics if x < y+k goto L (blt x y k L) for constant k and target address L embedded in the instruction. (Those are enough for computable functions, i.e., 'programs'; c.f., the programming language Fractran [11].)

A challenger readies a sequence of maximal entropy compilations $C[P_n]$ of P with the *n*th being for a *n*-bit platform as target, and P having been partially or completely unrolled as P_n with no loops or branches in the first 2^n (i.e., superpolynomially many) machine code instructions. If the program predictably ends before then, it is to be unrolled completely. These are compilations of the same program P all with the same end-to-end semantics that could be produced entirely automatically (the constants in P can be expressed exactly in the *n* bit words available on the platform, for all *n* considered). The observer is invited to apply their method f and predict the chosen bit b in the traces T_n . By hypothesis, $f(n, T_n)$ is correct with probability at least B with B > 1/2 as $n \to \infty$.

Let N be such that for $n \ge N$ the word of interest for the observer in the trace is produced by the unrolled part, that is, $2^N \ge M$. Consider the program Q that is P_N with the instruction that produces the word of interest changed from $x \leftarrow y+k$ to $x \leftarrow y+k'$ where k' = k+1, so the word is written in Q to 1 more than in P_N . But also all instructions in P_N that read that register or memory location x before it is written over again are changed in Q to compensate. That is, an instruction $z \leftarrow x+k_1$ in P_N is changed to $z \leftarrow x+k'_1$ in Q, where $k'_1 = k_1-1$; an instruction if $x < z+k_2$ goto L in P_N is changed to if $x < z+k'_2$ goto L in Q, where $k'_2 = k_2-1$. (There are only a finite number of instructions to change, as P_N is finite, but they may appear infinitely many times in the trace.)

Then the trace of Q is the same as the trace of P_N (and P_n , for $n \ge N$ except for that one word and its least significant bit. Moreover, the traces from the code C[Q] are each of them traces that may have been produced from code $C[P_N]$ (and/or any P_n for $n \ge N$) since the change from P_N to Q is one that the compiler may make, i.e. $Q = C[P_N]$ is possible.

Then the method f(n, -) applied to a trace of C[Q] on a platform with an n bit word produces the same result as when applied to a trace of $C[P_N]$ (and $C[P_n]$ for $n \ge N$), and so must produce the right result for P with probability at least B > 1/2 for n large enough. But that is the wrong result for

TABLE I TRACE FOR ACKERMANN(3,1)

	instruction				trace updates
36 37 38	beq t0 addi t0	zer t1	2	-86921031 -327157853 240236822 -1242455113	
41					t1 ← -1902505258 1242455113 1902505258 t0 = -17347613130
55 56		sp al zer tl		-1734761313 800875856 -915514235 -1175411995 259897760 11161509	sp ← 1687471183 t0 ← -915514234
	addi v0 jr ra P	t0		42611675	$v0 \leftarrow 13$ # result

	Legend		
op. fields	semantics	register	use
	$r_0 \leftarrow r_1 + k$	a0,a1,	function argument
	$pc \leftarrow pc + i$	pc	program counter
beq $r_1 r_2 i$	$ifr_1 = r_2 then pc \leftarrow pc + i$	ra	return address
jr r	$pc \leftarrow r$	sp	stack pointer
xor $r_0 r_1 r_2$	$r_0 \leftarrow (r_1 + k_1) (r_2 + k_2) - k_0$	t0,t1,	temporaries
$k_1 k_2 k_0$)	v0,v1,	return value
i program count increment, k instruction constant, r content of r			

Q. So the method f(n, -) produces the right result for Q with probability 1 - B < 1/2 for large enough n, contradicting the hypothesis, so the hypothetical method f does not exist.

The result for $m \ge 1$ follows because if the observer had a method f that made predictions at m>1 points in the trace, then the method g that throws away the m-1 predictions of f's at unwanted points would work as a method for m=1, and the proof rules that out. Similarly for more than one bit.

The argument is not deep, as the empty program case shows, but it is hard to codify and should increase understanding of this area. The chaotic compiler constructions are reminiscent of Yao's *garbled circuits* [12], at n bits and with recursion.

IV. IMPLEMENTATION

A prototype C compiler http://sf.net/p/obfusc covers ANSI C and GNU C extensions, including statements-as-expressions and expressions-as-statements, gotos, arrays, pointers, structs, unions, floating point, double integer and floating point data. It is missing **longjmp** and efficient strings (**char** and **short** are the same size as **int**). It is intended for use in encrypted computing [13], [14], [15], [16], [17], [18], an emerging processor technology in which inputs, outputs, and all intermediate values in registers and memory are in encrypted form. Because good encryption is one-to-many, and addresses are data like any other, many physically different bit sequences (the 'ciphertext') represent the address intended by the programmer (the 'plaintext'), and platforms exhibit hardware aliasing at every memory access, providing a testbed for theory and practice.

TABLE II

TRACE FOR SIEVE SHOWING EXTRA BITS (RIGHT) IGNORED IN ARITHMETIC BUT SIGNIFICANT IN MEMORY ADDRESSING. STACK READ LINES GRAY, ADDRESS BASE RED, DISPLACEMENT VIOLET.

PC in	struction				trace updates extra bits
22340 add	di t1 sp	-418452205	t1	\leftarrow	-877254954 1532548040
22360 bn	e t0 t1	84			
22384 add	di tl sp	-407791003	t1	\leftarrow	-866593752 1532548040
22404 lw	t0 866	593746(t1)	t0	\leftarrow	-866593745 1800719299
22424 add	di t0 t0	-1668656853	t0	\leftarrow	1759716698 1081155516
22444 b	540				
22988 ad	di t1 zer	1759716697	t1	\leftarrow	1759716697 1325372150
23008 bn	e t0 t1	44			
23128 ad	di t0 sp	-1763599776	t0	\leftarrow	2072564771 -1935092797
23148 lw	t0 -20	72564772(t0)	t0	\leftarrow	2072564779 -1773201679
23168 add	di t0 t0	1723411350	t0	\leftarrow	-498991167 -981581771
23188 add	di t0 t0	-1862832992	t0	\leftarrow	1933143137 - 1629507929
00000 1		-1933143130			-
23208 add	αι νυ τυ	-1933143130	v0	~	7 1680883739
 23272 jr STOP	ra				

See Table I for Legend.

A trace⁴ of the Ackermann function⁵ [19] compiled by the compiler is shown in Table I. The Ackermann function is the most computationally complex function possible, increasing in the degree of complexity required to calculate it (polynomial, exponential, super-exponential, etc.) with each increment of the first argument. It is not practically possible to 'cheat' in the calculation, and the compilation exercises the basic code constructs (function call, conditional, arithmetic, etc.), and the calculation is very sensitive, so it is a stiff test. The trace illustrates how the compiler's variation of the delta offsets for register content through the code results in randomly generated constants embedded in the instructions and randomly offset runtime data – until the result is returned with offset 0.

Running a Sieve of Eratosthenes program⁶ for primes is a delicate test of memory-oriented programming. The final part of the trace is shown in Table II with two stack reads in gray and the address base in red, with the address displacement (in the instruction) in violet. The trace shows 'extra bits' that form part of the data (and addresses) but which are not used in the arithmetic, in order to generate hardware aliasing. The extra is formed deterministically by hashing the extra bits and visible data of the operands to each arithmetic operation.

V. CONCLUSION

This paper has described the compilation of imperative source for a platform that has hardware aliasing with hidden determinism. The technique depends on the compiler controlling the address displacement and address base for load and store instructions so that they are always the same for repeat accesses to the same memory location. That uses saved copies or repeats earlier calculations exactly. The technique is extended to also generate as well as compensate for random displacements in the addressing, and extended again to generate and compensate for random displacements in the data content of all registers and memory (called chaotic compilation). A sketch proof is included to show this technique results in object code where it is difficult to discover the programmer's intention at any points of the runtime trace, while maintaining the semantics. I.e., the compiled code is technically 'obfuscated'.

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⁴Initial and final content offset deltas are set to zero here, for readability. ⁵Ackermann C code: **int** A(**int** m,**int** n) { **if** (m == 0) **return** n+1; **if** (n == 0) **return** A(m-1, 1); **return** A(m-1, A(m, n-1)); }.

⁶Sieve C code: int S(int n) { int $a[N]=\{[0...N-1]=1,\}$; if (n>Nn<3) return 0; for (int i=2; i<n; ++i) { if (!a[i]) continue; for (int j= 2*i; j<n; ++j) a[j]=0; }; for (int i=n-1; i>2; --i) if (a[i]) return i; return 0; }.