Crack resistance criterion of plane stress RC elements with prestressed reinforcement

Vitalij Kolchunov and Tatyana Iliushchenko

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V I Kolchunov [1], **T A Iliushchenko**[1]

[1] Department of Unique Buildings and Structures,South West State University, street 50 let Oktyabrya, 94, Kursk, 305040, Russia

E-mail: tatkhalina93@yandex.ru

Abstract. An analytical condition for crack resistance of a specific plane stress reinforced concrete element with prestressed reinforcement is considered in this article. The specific concrete element is reinforced with orthogonally located prestressed rods that coincide with the directions of the coordinate axes. The crack resistance condition is constructed by generalizing of G.A. Geniev theory of plasticity of concrete and reinforced concrete on the region of existence of tensile stresses: "tension-tension" and "compression-tension". A graphical interpretation of this condition is presented for a plane stressed element depending on the angle between the main force and the coordinate axes. As an example, the crack resistance condition of the support zone of a prestressed concrete beam of a monolithic reinforced concrete frame is considered.

1. Introduction

Whereas it is necessary to protection of buildings and structures from progressive collapse, which is provided for by the requirements of regulatory documents of various countries, including Russia, research is being conducted and various proposals are being developed to solve this problem. As such proposals, in relation to the RC frames of multi-storey buildings, various protection methods are recommended, which include increasing the cross-sections of the load-bearing elements of the building frame [1, 2], intensity and reinforcement schemes [3, 4], setting up additional elements in the form of a links system [5, 6], the exclusion of one-way bonds in compounds of elements [7], etc. One of the effective methods of such protection can be the application of prestressing beams over the first, and if necessary, over other floors of the building. In this regard, there is a need to solve a number of theoretical problems, one of which is the task of determining the crack resistance condition of a plane stressed RC element with prestressed reinforcement.

2. Creation of crack resistance criterion

An analytical form of the crack resistance criterion for a RC element with prestressed reinforcement can be constructed by generalizing the strength condition of G.A. Geniev the theory of plasticity of concrete and reinforced concrete [8] on the region of existence of tensile stresses: “tension-tension” and “compression-tension”. When constructing analytical dependencies, the following are accepted as initial hypotheses:

- the specific plane stressed concrete element is reinforced with orthogonally located prestressed rods that coincide with the directions of the coordinate axes of the XOY system (Fig. 1);
- only axis normal stresses in reinforcing rods and, in the general case, the initial effect is taken into account in a plane stress state. This assumption is due to the fact that before the formation of cracks in a characteristic element under the action of tangent forces, the dowel action in reinforcing rods is miniscule;
- the forces in the tensile reinforcement reach the forces of crack formation during crack formation in concrete of a plane stressed element equal to the total force acting in the section under consideration in the direction normal to the crack.

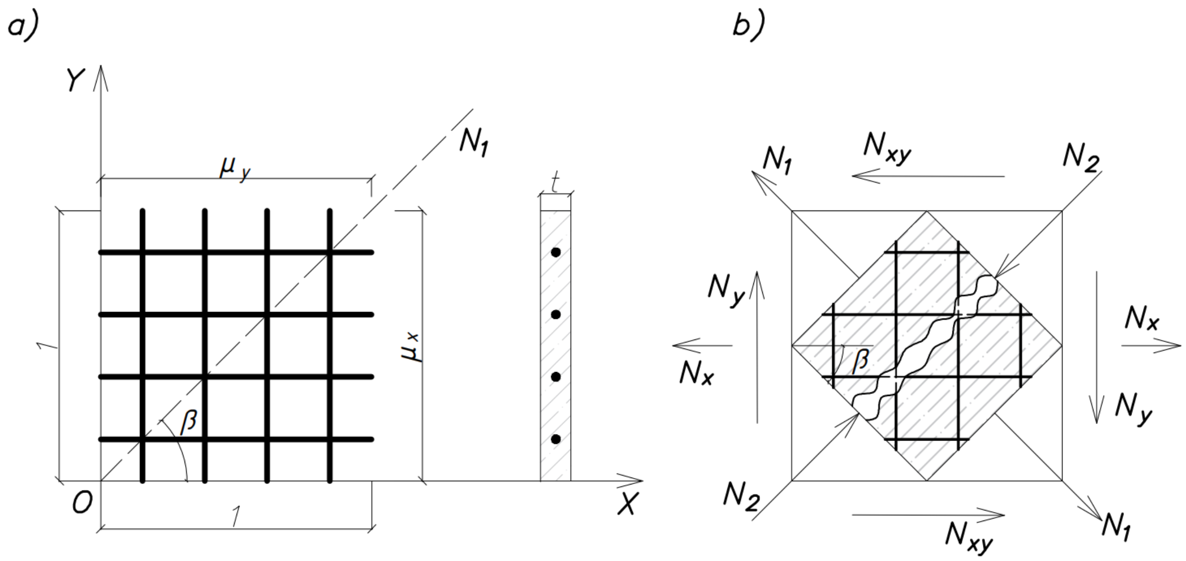

Figure 1. Scheme of a characteristic prestressed element in the axes of the XOY (a) and in the axes of the main forces (b).

When constructing the analytical crack resistance conditions with positive stresses, we will take compressive stresses as negative. We express the total axial forces in a characteristic RC element of unit dimensions N_i through the critical forces in concrete $\overline{N}_i$ and the forces reduced to concrete equivalent to the forces in the reinforcement at the cracking moment in concrete $\mu_i N_{s,crc}$:

$$\left.\begin{aligned} N_x &= \overline{N}_x + \mu_x N_{s,crc}; \\ N_y &= \overline{N}_y + \mu_y N_{s,crc}; \\ \tau_x &= \overline{\tau}_x; \tau_y = \overline{\tau}_y. \end{aligned}\right\} \quad (1)$$

We will assume that the value μ_i can be both positive and negative, determining the sign of the force in the reinforcement. Obviously, for the signs in concrete to match the corresponding reduced reinforcement stress, it is necessary that $|N_i| \geq |\mu_i| N_{s,crc}$.

The crack resistance condition of RC can be represented by analogy with the plasticity condition [8] for a plane stress state in the XOY axes:

$$\overline{N}_x^2 + \overline{N}_y^2 - (N_x \cdot N_y) + 3N_{xy}^2 - (R_b - R_{bt})(\overline{N}_x + \overline{N}_y) - R_b \cdot R_{bt} \cdot t = 0 \tag{2}$$

where t is the thickness of the specific plane stressed element (see Fig. 1).

Since the crack resistance criteria of a typical plane stressed element in the general case, it is convenient to present the analysis of the directions of crack formation in the components of the main forces N_1 and N_2,, we express condition (3) in terms of the N_1 and N_2, using the known dependences of the theory of elasticity between the components of the forces when the axes rotate:

$$\left.\begin{aligned} N_x &= \frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2}\cos 2\beta; \\ N_y &= \frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2}\cos 2\beta; \\ N_{xy} &= \frac{N_1 - N_2}{2}\sin 2\beta, \end{aligned}\right\} \tag{3}$$

where β is the angle between the direction of the larger main axial force N_1 and the positive direction of the X axis (see Fig. 1):

Substituting (3) into (2), we obtain the crack resistance condition of a specific plane stressed element in the components of the main forces:

$$\begin{aligned}
&(\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2}\cos\beta)^2 + (\frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2}\cos\beta)^2 - (\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2}\cos\beta) \\
&(\frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2}\cos\beta) - t(R_b - R_{bt})(\frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2}\cos\beta + \frac{N_1 + N_2}{2} + \frac{N_1 - N_2}{2}\cos\beta) + \\
&+ 3(\frac{N_1 - N_2}{2}\sin\beta)^2 - \mu_x N_{s,crc}((N_1 + N_2) + (N_1 - N_2)\cos\beta) - \\
&- \frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2}\cos\beta) - \mu_y N_{s,crc}((N_1 + N_2) + (N_1 - N_2)\cos\beta) - \frac{N_1 + N_2}{2} - \frac{N_1 - N_2}{2}\cos\beta) + \\
&+ N_{s,crc}{}^2(\mu_x{}^2 - \mu_x\mu_y + \mu_y{}^2) + t(R_b - R_{bt})(\mu_x + \mu_y)N_{s,crc} - t \cdot R_b \cdot R_{bt} = 0
\end{aligned}$$

$$\begin{aligned}
&N_1^2 - N_1 N_2 + N_2^2\left[t(R_b - R_{bt}) + \frac{\mu_x + \mu_y}{2}N_{s,crc}\right](N_1 + N_2) - 3\frac{\mu_x + \mu_y}{2}N_{s,crc}(N_1 + N_2)\cos\beta \\
&+ (\mu_x^2 - \mu_x\mu_y + \mu_y^2)N_{s,crc}^2 + t(R_b - R_{bt})(\mu_x + \mu_y)N_{s,crc} - t \cdot R_b \cdot R_{bt} = 0.
\end{aligned} \tag{4}$$

Analyzing the crack resistance condition (4) for the considered plane stressed element, it will be seen, on conditions that $\mu_x \neq \mu_y$ the types of the crack resistance criterion depend on the angle made by the direction of the N_1 with the direction of reinforcement. In general, when $\mu_x \neq \mu_y$ and $|\beta| \neq \frac{\pi}{4}$ the crack resistance condition (4) in coordinate system $N_1 N_2$ represents an ellipse whose main axes do not pass through the origin and make angles $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ with the direction of the force. At an $|\beta| = \pm\frac{\pi}{4}$, the large major axis of the ellipse passes through the origin of the system $N_1 N_2$.
Let us analyze the specific points of the crack resistance criterion (4), assuming in the general case that crack formation in a specific element can arise from the main tensile stresses in the «tensile-tensile», «tensile-compressive» regions (see Fig. 1b).

3. Uniaxial state of stress

Consider the crack resistance condition of a specific prestressed concrete element during reinforcement only in the direction of the main tensile forces $\beta = 0; N_2 = 0; \mu_y = 0$. For this case, condition (4) can be written as

$$N_1^2\left[t(R_b - R_{bt}) + 2\mu_x N_{s,crc}\right]N_1 + \mu_x^2 N_{s,crc}^2 + t(R_b - R_{bt})\mu_x N_{s,crc} - t \cdot R_b \cdot R_{bt} = 0. \qquad (5)$$

For the case of uniaxial compression at $\mu_x = \mu > 0$ equation (5) determines the relation:

$$N_1^b = tR_b + \mu N_{s,crc}, \qquad (6)$$

and in the case of uniaxial tension at $\mu_x = -\mu$:

$$N_1^{bt} = -tR_{bt} + \mu N_{s,crc}, \qquad (7)$$

where μ is the absolute value of the reinforcement coefficient/

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