Mathematical Modeling for Crude Oil Export Terminal Scheduling and Tank-Farm Operations

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August 6, 2024

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Abstract. In the crude oil shipping industry, crude oil terminals play a pivotal role in maintaining the global economy by ensuring a sustainable supply of crude oil to the world. In this paper, a novel mathematical model that captures the crude oil terminal scheduling is proposed. The proposed model schedules customers' oil carriers as well as allocates adequate storage capacity in the tanks for multiple crude oil products to maintain a safe level of operation. The model aims to attain customers' satisfaction by providing an export schedule that meets customers' preferences. Specifically, the model's primary objective is to minimize the deviation from the customers' preferred loading date. In addition to that, controlling the tank inventory level and the number of tanks that switched their service are our secondary objectives. A weighted sum approach to handle multiple objectives is proposed in this work. A numerical case study that illustrates the applicability of the proposed model is provided in this paper. Finally, the paper concludes with discussion of the results and future research directions.

Keywords: Crude Oil Scheduling, Tank-Farm Operations, Mathematical Modeling, Optimization

1 Introduction

In the crude oil shipping industry, crude oil terminals play a pivotal role in maintaining the global economy by ensuring a sustainable supply of crude oil to the world. This paper will discuss how to integrate the oil carriers scheduling with tank-farm inventory management as a Mixed Integer Linear Programming (MILP) model. our goal is to build a dynamic inventory capacity multi-product single-location inventory model that is integrated into the oil carriers scheduling model. This model involves handling of various crude oil grades in a terminal with a continuous supply. Oil carriers must be scheduled to load a certain number of products with their scheduled quantities. Additionally, oil carriers must be loaded on specified days due to their commitments, or to avoid deviating significantly from the specified days. Moreover, the crude oil terminal has multiple capacitated resources such as: the number of berths to serve oil carriers in the same time, capacity of tanks in the tank-farm, number of tanks that have the ability to switch their service, and the supply and loading capacity for each crude grade.

2 Literature Review

To the best of our knowledge, there are not many papers in the literature that discuss this problem. A majority of the papers are concerned with vessel routing and berth allocation in marine supply chain topics. Specifically, many papers discussed the Berth Allocation Problem (BAP). In [5], the authors introduce a methodology to solve a discrete berth scheduling problem via hierarchical optimization and genetic algorithm. In [3], the authors formulate the BAP as an open shop scheduling problem, then solve it using MILP and Constraint Programming (CP). In [2] the authors demonstrate a solution approach to assist employees in preparing the ships dispatching schedule using a mathematical programming model. Other papers, in the literature discuss vessel routing problem for marine movement. The authors in [1], brought a real life problem for a petrochemical plant that ships product to customers using a few numbers of owned ships. The problem considers product availability and tugging operation to load the ships, then transport the product to a port of destination and return for the next trip. The problem is modeled as MILP model. An initial solution is obtained by a heuristic which is then inserted to the CPLEX solver to reduce the solution time. The authors in [8] introduce a two level model that assign a vessel to a task that consists of combination of product and destination, that maintain the demand at the destination. The second level assigns vessels to days to ensure no overlap between vessels operations. A vendor managed inventory system is formulated as a MILP model, which is also known as a maritime inventory routing problem in [6]. The model optimize he whole supply system considering ship movements and jetty allocations. Some papers discussed the tank-farm management as pipeline scheduling problem for example in [7] the authors present an MILP model for the tank farm inventory management for a single pipeline and multi-product scenario. In the paper, the authors discuss the pipeline product scheduling problem that maintains a safe inventory to satisfy all customers demand. The authors in [4] present a new MILP formulation for pipelines that schedules products in the pipelines with optimum sequence to avoid undesirable product intermixing while satisfying the demand on time.

In this work, the focus is on the day-to-day tactical scheduling decisions that are to be made at the crude oil terminal. The key decisions involve assigning oil carriers to a loading day, maintaining sufficient inventory level in the oil tanks, and reducing the service changes in the oil tanks. These decisions are captured through a novel mathematical model in this work. The primary objective is to minimize the deviation between the customers' preferred loading date and the berth assignment date. The tank inventory levels are controlled via soft and hard constraints. The number of tanks that switched their service is modeled as soft constraint. Therefore, the soft constraints form secondary objectives of the proposed model. A weighted sum approach to handle multiple objectives is proposed in this work.

The rest of the paper is organized as follows: In Section 3 a novel mathematical model for the crude oil terminal scheduling is proposed. Key assumptions, parameters and variables are defined in this section. Finally, the details of the mathematical model is presented. Section 4 illustrates the validity of the proposed model through a case study. Results are depicted to assert the applicability of the proposed model. Finally, in Section 5, we highlight the key achievements of the work and propose future research directions.

3 Crude Oil Terminal Scheduling (COTS) Model

An assignment model to schedule the oil carriers will be used as a basis. Each carrier will be assigned to a loading day, that should be as close as possible to its requested loading day. Hence the problem will have a discrete time representation, i.e. each day will be a slot that can take a number of oil carriers. Following subsections present assumptions on the model.

3.1 Assumptions

- The demand is always less than the supply including the opening inventory.
- Back-order is not allowed. Inventory levels should never go below zero.
- Inventory will be aggregated per product pool for all locations.
- The number of carriers to be scheduled in a day is half of the number of available berths, this is to eliminate the possibility of overlapping carriers on the same berth in case of unfinished carrier loading within a day.
- The impact of changing the loading capacity in two consecutive days is negligible.

3.2 Model Formulation

Indices:
i: Crude carrier index, $i = 1, 2, ...n$ and $i \in I$
j: Day index, $j = 1, 2, ...d$ and $j \in J$
k: Product index, $k = 1, 2, ...p$ and $k \in P$
l: Location index, $l = 1, 2, ...m$ and $l \in M$

Decision Variables:

$$X_{ij} = \begin{cases} 1, \text{ If crude carrier } i \text{ is scheduled on day } j \\ 0, \text{ Otherwise} \end{cases}$$

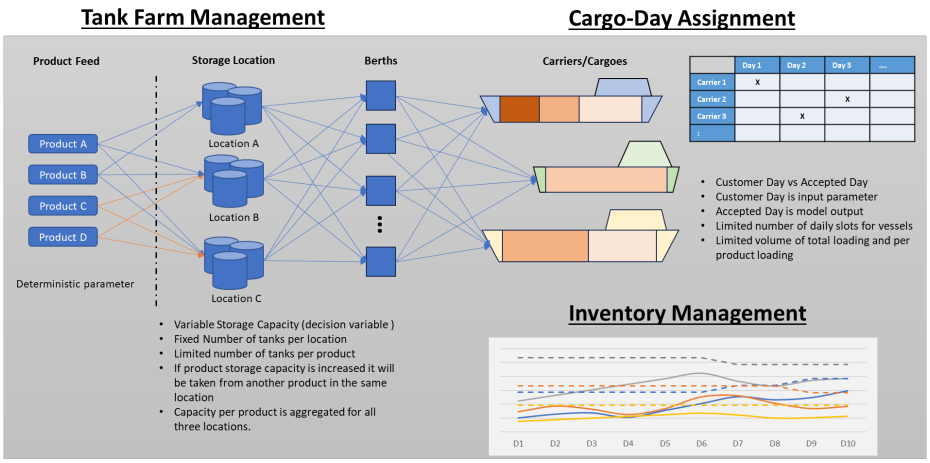

Fig. 1. Problem Schematic

T_{jkl} = Number of tanks in tankfarm l that stores product k in day j
r_{jk}^{+} = Volume of product k violating the soft maximum constraint for inventory in day j
r_{jk}^{-} = Volume of product k violating the soft minimum constraint for inventory in day j

Parameters:

D_j = day j in the planning horizon = 1, 2, 3, ... , d

N_i = Oil carrier i requested loading day

C_{ik} = Cargo size of product k to be loaded on carrier i

$$P_{ik} = \begin{cases} 1 \text{ , if carrier } i \text{ is asking for product } k \\ 0 \text{ , otherwise} \end{cases}$$

PM_{jk} = Maximum number of cargoes of product k that can be loading in day j

$AvrgC_k$ = Average cargo size for product k

$Mdel$ = Maximum number of days a carrier can be delayed

$Madv$ = Maximum number of days a carrier can be advanced

OI_{jk} = Opening inventory of product k in day j

F_{jk} = Product k feeding the terminal in day j

S_j = Maximum number of carrier to be scheduled in day j

$AvgT_{kl}$ = Average Tank size of product k in location l

$MaxT_{kl}$ = Maximum number of tanks that can store product k in location l

X_i^+ = Linearization for the absolute value function for carrier i day deviation

L_{jk} = Total product k loading in day j

Cap_{jk} = Storage capacity for product k in day j

I_{jk} = Inventory of product k in day j

TSC_{jkl} = Number of tanks added or taken out from product k service in location l in day j

TSC^+_{jkl} = Linear representation for the absolute value function for tanks services change

Objective Function:

$$\begin{aligned} Min \quad Z = m^+ * Mdel + m^{++} * Madv + \sum_{i=1}^{n} Pr_i * X_i^+ \\ + cr * \sum_{k=1}^{p}\sum_{j=1}^{d}(r^+_{jk} + r^-_{jk}) + w * \sum_{l=1}^{m}\sum_{k=1}^{p}\sum_{j=1}^{d} TSC^+_{jkl} \end{aligned} \tag{1}$$

Constraints:

$$\sum_{j=1}^{d} X_{ij} * D_j - N_i \leq X_i^+ \quad \text{and} \quad N_i - \sum_{j=1}^{d} X_{ij} * D_j \leq X_i^+ \tag{2}$$

$$\sum_{j=1}^{d} X_{ij} * D_j - N_i \leq Mdel \tag{3}$$

$$\sum_{j=1}^{d} X_{ij} * D_j - N_i \geq -Madv \tag{4}$$

$$\sum_{j=1}^{d} X_{ij} = 1 \quad \forall i \tag{5}$$

$$L_{jk} = \sum_{i=1}^{n} X_{ij} * C_{ik} \quad \forall j, k \tag{6}$$

$$I_{jk} = OI_{jk} + F_{jk} - L_{jk} \qquad for \quad j = 1 \;\&\; \forall k \tag{7}$$

$$I_{jk} = I_{j-1,k} + F_{jk} - L_{jk} \qquad for \quad j \geq 2 \;\; \& \;\; \forall k \tag{8}$$

$$I_{jk} \leq Cap_{jk} \quad \forall j,k \tag{9}$$

$$I_{jk} \geq 0 \quad \forall j,k \tag{10}$$

$$I_{jk} \leq 0.8 * Cap_{jk} + r^{+}_{jk} \quad \forall j,k \tag{11}$$

$$I_{jk} \geq 3 * AvrgC_{k} - r^{-}_{jk} \quad \forall j,k \tag{12}$$

$$\sum_{i=1}^{n} X_{ij} \leq S_{j} \quad \forall j \tag{13}$$

$$\sum_{i=1}^{n} X_{ij} * P_{ik} \leq PM_{jk} \quad \forall j,k \tag{14}$$

$$\sum_{k=1}^{p} L_{jk} + L_{j+1,k} \leq LC_{j} + LC_{j+1} \quad \forall j \tag{15}$$

$$Cap_{jk} = \sum_{l=1}^{m} T_{jkl} * AvgT_{kl} \quad \forall j,k \tag{16}$$

$$T_{jkl} \leq MaxT_{kl} \quad \forall j \tag{17}$$

$$\sum_{k=1}^{p} T_{jkl} = ATanks_{jl} \quad \forall j,l \tag{18}$$

$$\frac{|T_{jkl} - T_{j-1,kl}|}{2} = TSC_{jkl} \quad \forall j,k,l \tag{19}$$

$$TSC_{jkl} \leq TSC^{+}_{jkl} \quad and \quad -TSC_{jkl} \leq TSC^{+}_{jkl} \quad \forall j,k,l \tag{20}$$

3.2.1 Crude Oil Carriers to Day Assignment

Equation (21) calculates the deviation from the requested loading day.

$$|\sum_{j=1}^{d} X_{ij} * D_{j} - N_{i}| \quad \forall i \tag{21}$$

However, Equation (21) is nonlinear equation and it can be linearized by introducing X_i^{+} as shown in Equation (2). In addition to that, there is a maximum number of days that an oil carrier can deviate, *delayed or advanced*, is estimated as an upper limit of $Mdel$ and a lower limit of $Madv$. Equation (3) and Equation (4) show this implementation.

Noting that $\sum_{j=1}^{d} X_{ij} * D_j - N_i$ can be positive or negative, thus its lower bound will be greater than the negative of $Madv$ as shown in Equation (4). Finally, the last constraint, Equation (5), will ensure all carriers are assigned to a loading day only once.

3.2.2 Inventory Capacity Constraints

Variable I_{jk} calculates the inventory for product k in day j as shown in Equations 7 and 8. Where L_{jk} is the total loaded quantity of product k in day j as shown in Equation (6) and F_{jk} is the amount of product k feeding the terminal on day j. Moreover, parameter OI_k is the opening inventory for product k i.e. it will be considered only when $j = 1$.

Variable Cap_{jk} is defined as the storage capacity for product k in day j. It is a variable since it will be dynamically changed as shown in Equation (16). Now, I_{jk} is maintained based on two kinds of constraints, soft constraints that are allowed to be violated with a penalty, and hard constraints related to the physical storage capacity. The hard constraints are straight forward as shown in Equations (9) and (10). To address the soft constraints, we introduce two dummy variables r^+_{jk} and r^-_{jk} to allow the model to violate upper and lower bounds respectively with a penalty cr that will be included in the objective function. Here, Equation (11) is related to the soft max constraint and it will be based on 80% of the available capacity. Additionally, Equation (12) is related to the soft min constraint and it is based on three times the average cargo size of product k, during the planning horizon. $AvrgC_k$ is used to calculate the average cargo size in the planning horizon as shown in Equation (22).

$$AvrgC_k = \frac{\sum_{i=1}^{n} C_{ik}}{\sum_{i=1}^{n} P_{ik}} \tag{22}$$

where $\sum_{i=1}^{n} C_{ik}$ is the total volume of product k planned to be exported and $\sum_{i=1}^{n} P_{ik}$ is the total number of cargoes of product k planned to be loaded for all available carriers.

3.2.3 Crude Carrier and Cargo Scheduling Constraints

S_j is defined as the maximum number of carriers allowed to be scheduled to load on the same day. The number of crude carriers to be assigned on day j is shown in Equation (13).

Additionally, since each loading system can handle one product at the same time, and due to the available capacity of the loading systems, limiting the number of cargoes of the same product to be loaded on the same day is needed. Let P_{ik} be an input such that:

$$P_{ik} = \begin{cases} 1 \text{ , if carrier } i \text{ is asking for product } k \\ 0 \text{ , otherwise} \end{cases}$$

and PM_{jk} to be maximum number of cargoes lifted of product k on day j. Equation (14) shows how to limit the number of cargoes of the same product to be loaded on the same day.

3.2.4 Loading Capacity Constraints

Crude oil terminals load crude carriers through loading lines using pumps. Those loading lines have a capacity to transfer the crude oil from the tanks to the carrier, based on their size and pumping rate. The logic can be formulated by introducing LC_j (the maximum loading capacity for the crude oil terminal for day j), and using L_{jk} as defined in Equation (6). However, since large carriers usually take more than one day and less than two days to be fully loaded, and since the model is based on a discrete-time representation, keeping track of the loaded quantity for two consecutive days and maintaining it below the loading capacity for two days is required. Equation (15) is an implementation of this logic.

3.2.5 Storage Capacity Constraints

In this dynamic system, storage capacity can be planned to be altered as needed, based on the storage tanks to product allocation on day to day basis. The crude grades are handled in multiple tank farms, each of which has a different capacity, number of tanks, and specific crude grades that can be handled there.

In Equation (16), Cap_{jk} is defined as the assigned storage capacity for product k in day j, and $AvgT_{kl}$ is the average tank size located in tank farm l that can handle product k. The quantity $T_{jkl} \times AvgT_{kl}$ provides a good indication of the calculated capacity at location l to store product k on day j.

Also, there are a limited number of tanks at tank farm l that can handle product k. Let $MaxT_{kl}$ be defined as the maximum number of tanks, at tank farm l that can handle product k. Equation (17) limits the number of tanks assigned to product k on day j in tank farm l to be less than or equal $MaxT_{kl}$. Finally, all available tanks must be used, Equation (18) ensures the usage of all available tanks.

3.2.6 Tracking Number of Tanks Service Change

Although the system allows for change of the tank service when needed, it is not preferable and the number of tank service changes needs to be minimized. Equation (19) captures the number of tanks' service changes between two consecutive days utilizing variable TSC_{jkl}. Additionally, as the change may include adding or removing a tank from the service of product k, TSC_{jkl} will represent the absolute difference in the number of tanks required to serve product k at tank farm l between days j and $j-1$. Moreover, Equation (19) shows the calculation of the number of tanks which avoids double counting the number of tanks' service reassignment. Because once the count of tanks that serve a product increases by one the count of tanks that serve another product will decrease by one. This means the total number of service changes will be two, whereas it is only one tank that changed its service. This will not play a big role in the model since it is only dividing by a constant, however, it is more practical as it will indicate the number of tanks that changed their service. Furthermore, in order to linearize the equation, one additional variable TSC^{+}_{jkl} is introduced with two additional constraints as shown in Equation (20).

3.2.7 Objective Function

Following penalties and metrics are defined as part of the objective function:

- Pr_i: Priority of carrier i, the higher the priority the less the deviation from the requested date.
- w: Penalty for changing tank service.
- cr: Penalty for violating soft inventory limits.

The objective function will be as shown in 23:

$$Min \quad Z = \sum_{i=1}^{n} Pr_i * X_i^+ + w * \sum_{l=1}^{m}\sum_{k=1}^{p}\sum_{j=1}^{d} TSC_{jkl}^+ + cr * \sum_{k=1}^{p}\sum_{j=1}^{d}(r_{jk}^+ + r_{jk}^-) \tag{23}$$

which minimizes the deviation of the carriers' requested loading date, the inventory limits violation, and the number of tanks that switched their service. However, the objective presented in Equation (23) is not unified, since each component is measured based on different units. The first component which is related to the carriers' deviation days is measured in days, the second component which is related to tanks service change is in the number of tanks, and finally, the last component which is related to the violation of the inventory soft limits is in volume.

Unifying the objective function can be easily done by converting the three sections to volumes, by multiplying the deviation days by the carrier cargo size, and multiplying the average tank size for product k by the number of tanks service change as shown in Equation (24).

$$\begin{aligned} Min \quad Z = \sum_{k=1}^{p} C_{ik} * \sum_{i=1}^{n} Pr_i * X_i^+ + w * \sum_{l=1}^{m}\sum_{k=1}^{p}\sum_{j=1}^{d} AvgT_{kl} * TSC_{jkl}^+ \\ + cr * \sum_{k=1}^{p}\sum_{j=1}^{d}(r_{jk}^+ + r_{jk}^-) \end{aligned} \tag{24}$$

Although, the units are unified, notice that the objective presented in Equation (24), will give more priority to carriers' with larger volumes to be scheduled first. This logic is not in alignment with the business requirements, as the volume of crude carrier and the size of the tank do not influence in the prioritization of the carriers or the tanks.

3.2.8 Robust Approach Against Infeasibility: After experimenting with a number of datasets, it was found the model cannot schedule all carriers within the predefined limits, $Mdel$ & $Madv$. To counter this issue, the model minimizes the maximum days delayed and advanced, hence including the upper and lower limits, $Mdel$ and $Madv$ in the objective function with a penalty, m^+, for $Mdel$ and a high penalty, m^{++}, for $Madv$ (since accepting advancing the loading date may not be acceptable to the customers). Hence, the final objective function will be as shown in Equation (1).

4 Case Study

A numerical case study is used for evaluating the performance of the model. The case study consists of 4 products, 3 tank-farms with aggregated product pool, and 30 days planning horizon.

The case study involved several crude carriers to be scheduled at the terminal with a different cargo mix, and different preferred loading day. Additionally, for each product, a daily feed rate to the terminal is provided with the opening inventory. Furthermore, the number and the capacity of the available storage tanks are considered along with all other loading capacity limits.

The proposed mathematical model is modeled in GAMS[4] and solved via CPLEX[5] solver. The terminating criteria for the solver are either reaching to a global optimal solution or reaching a time limit of 3 hours. The model is executed on a Laptop PC with Intel Core i7 Processor (4x 2.5 GHz) and 12 GB RAM.

4.1 Inventory Profiles

Inventory profiles obtained from the model's solution are illustrated in Figures 2 to 5. All inventory physical limits are respected, and the model maintains the inventory levels below the soft maximum limit of 80% of the capacity. Also, the graphs show tanks allocation for each product at each tank-farm, to maintain minimum loading days deviation.

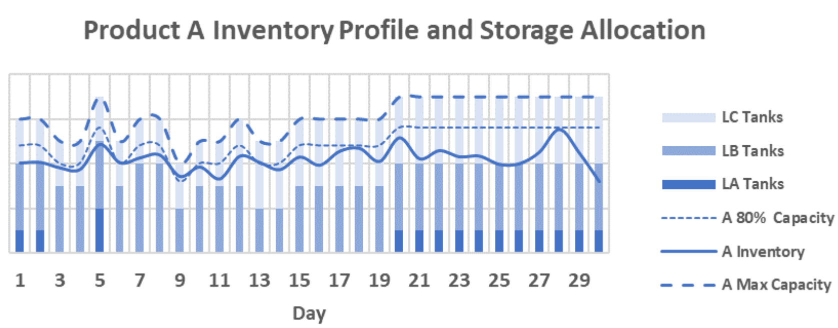

Fig. 2. Inventory Profile and Storage Allocation for Product A

4.2 Loading Day Deviation from Requested Day

The main objective of the model is to minimize the deviation of the actual loading day from the requested loading day. The model achieves to schedule

[4] https://www.gams.com/
[5] https://www.ibm.com/products/ilog-cplex-optimization-studio

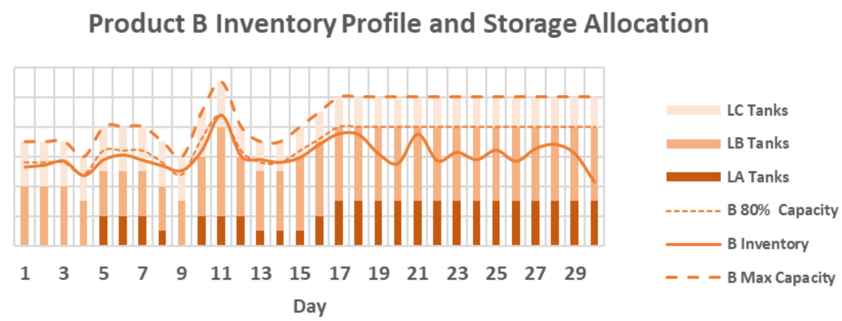

Fig. 3. Inventory Profile and Storage Allocation for Product B

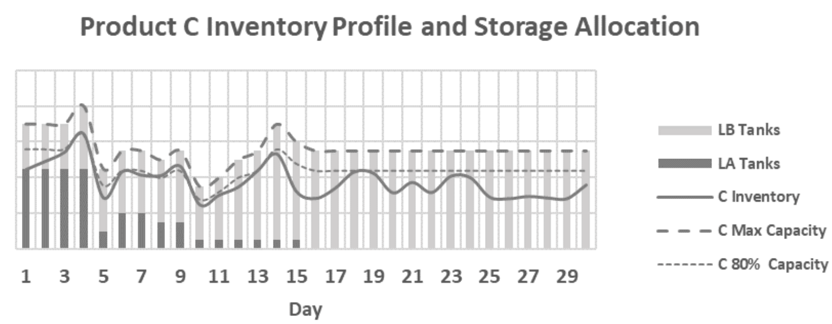

Fig. 4. Inventory Profile and Storage Allocation for Product C

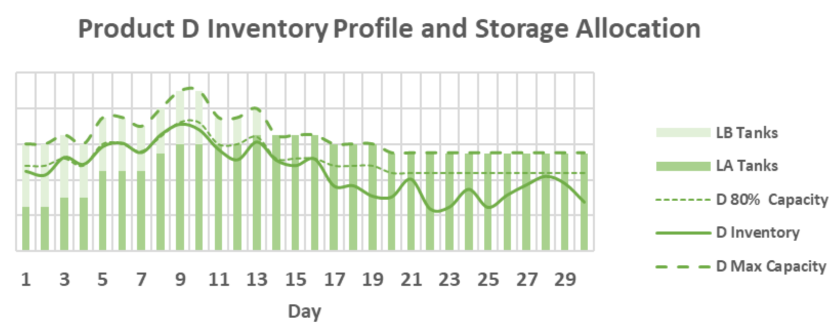

Fig. 5. Inventory Profile and Storage Allocation for Product D

68% of the carriers to load within 2 days of their requested loading day, 9%

within 3 to 4 days, 10% within 5 to 6 days and 13% within 7 days deviation as illustrated in Figure 6.

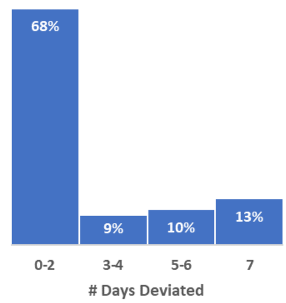

Fig. 6. Loading Day Deviation

5 Conclusion

Although the model produced a feasible solution, there is a 7% optimally gap, *to the relaxed problem*. Additionally, the result shows in Figure 6 that the number of carriers loading day deviation is increasing instead of decreasing. This can be a result of the conflicting terms within the objective function, and the imbalanced costs associated with each term. A better method to handle this problem is to solve it as a multi-objective optimization problem. Also, a continuous time representation can be explored in order to minimize the number of integer variables that can reduce the computational time significantly.

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