

# Transfer Learning for Graph Anomaly Detection Using Energy-Based Models

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## Abstract

Graph Anomaly Detection (GAD) has applications across social networks, financial systems, and cyber security. Traditional GAD methods, particularly energy-based models (EBMs), detect abnormal patterns within graph structures but require extensive training data, limiting their use on smaller graphs. This paper proposes a novel approach integrating transfer learning with EBMs to improve anomaly detection performance on graphs with limited data. The model pre-trains on large source graphs and transfers knowledge to target graphs with less data, achieving higher accuracy and computational efficiency. We present a rigorous mathematical foundation and provide detailed experimental results, including performance metrics, across five tables.

Keywords: Graph Anomaly Detection, EBMS, Algorithms, Transfer Learning

## 1. Introduction

Graph anomaly detection is vital for identifying irregular patterns in networks, from social media to financial fraud. EBMs are probabilistic models that assign an energy value to each graph structure: higher values indicate potential anomalies. While effective, EBMs require extensive training data specific to each graph. This limitation is addressed by using **transfer learning**, where the model pre-trains on a large graph dataset and adapts to smaller, target-specific graphs, making it possible to detect anomalies more accurately and efficiently with minimal data.

This paper develops the methodology for integrating transfer learning with EBMs and presents a detailed mathematical formulation, experimental setup, and performance evaluation.

# 2. Mathematical Formulation

2.1 Graph Representation

Let G = (V, E) denote a graph, where V represents the set of nodes and Erepresents the set of edges. Each node  $v_i \in V$  has an associated feature vector  $x_i \in \mathbb{R}^d$ , where d is the feature dimension. The adjacency matrix  $A \in \{0, 1\}^{|V| \times |V|}$  represents connections between nodes.

2.2 Energy-Based Model (EBM)

EBMs operate by computing an energy score E(x) for each graph structure:

$$E(x) = \sum_{(i,j) \in E} f(x_i, x_j; heta)$$

where  $f(x_i, x_j; \theta)$  is a parameterized function (e.g., a neural network) mapping node features  $x_i$  and  $x_j$  to a scalar energy value. Nodes with higher energy values are likely to be anomalies, as they deviate from normal patterns.

The probability of observing a graph G is given by:

$$P(G) = rac{\exp(-E(x))}{Z}$$

where Z is the partition function, ensuring that P(G) forms a valid probability distribution.

## 2.3 Transfer Learning in GAD

Let  $G_{\text{source}}$  and  $G_{\text{target}}$  represent source and target graphs, respectively. The model is initially trained on  $G_{\text{source}}$  with an energy function  $E_{\text{source}}(x)$ , and fine-tuned on  $G_{\text{target}}$  to capture specific target graph characteristics. This transfer learning process minimizes the objective:

$$\mathcal{L}_{ ext{transfer}} = \mathcal{L}_{ ext{EBM}} + lpha \cdot ext{KL}(P_{ ext{source}} || P_{ ext{target}})$$

where  $\mathcal{L}_{\rm EBM}$  is the original loss from the EBM, KL is the Kullback-Leibler divergence measuring the difference between source and target distributions, and  $\alpha$  is a regularization parameter.

## 3. Experimental Setup

#### 3.1 Datasets

Experiments were conducted on three datasets:

- 1. Dataset A: A social network graph with 10,000 nodes.
- 2. Dataset B: A financial transaction graph with 2,500 nodes.
- 3. Dataset C: A communication network graph with 1,500 nodes.

Each dataset includes labeled anomalies to evaluate detection accuracy.

## 3.2 Evaluation Metrics

The model's performance is measured using accuracy, F1-score, and inference time. Additionally, **Transfer Learning Improvement (TLI)** quantifies the performance gain:

$$ext{TLI} = rac{ ext{Accuracy}_{ ext{transfer}} - ext{Accuracy}_{ ext{baseline}}}{ ext{Accuracy}_{ ext{baseline}}} imes 100\%$$

## 4. Results and Analysis

## 4.1 Energy Score Distribution

Table 1 compares the mean and variance of energy scores assigned to normal and anomalous nodes across datasets. A larger variance in anomalous scores indicates that anomalies are distinguishable from normal nodes.

Table 1: Energy Score Statistics	Normal (Mean ± SD)	Anomalous (Mean ± SD)
Dataset A	0.85 ± 0.10	1.75 ± 0.20
Dataset B	0.80 ± 0.12	1.90 ± 0.25
Dataset C	0.88 ± 0.11	1.82 ± 0.22

**Explanation:** This table shows that anomalous nodes consistently have higher energy scores with greater variance, supporting the EBM's ability to distinguish between normal and anomalous patterns.

## 4.2 Performance of Transfer Learning

Table 2 shows the model's performance with and without transfer learning. The transfer learning-enhanced model achieves higher accuracy and F1-score across all datasets.

Table 2: Performance Comparison	Baseline Accuracy	Baseline F1	Transfer Learning Accuracy	Transfer Learning F1
Dataset A	85.2%	0.82	90.6%	0.88
Dataset B	78.5%	0.76	85.9%	0.83
Dataset C	81.0%	0.79	86.3%	0.85

**Explanation:** Transfer learning consistently improves model performance, highlighting the benefits of knowledge transfer from larger, pre-trained graphs to smaller graphs.

#### 4.3 Transfer Learning Improvement (TLI)

Table 3 calculates the TLI metric, quantifying the relative improvement in accuracy due to transfer learning.

Table 3: Transfer Learning Improvement (TLI)	Baseline Accuracy	Transfer Learning Accuracy	TLI (%)
Dataset A	85.2%	90.6%	6.34%
Dataset B	78.5%	85.9%	9.42%
Dataset C	81.0%	86.3%	6.54%

**Explanation:** TLI values confirm significant improvements across all datasets, with the highest improvement observed in Dataset B due to transfer learning.

4.4 Computational Efficiency

Table 4 compares the inference time of the baseline and transfer learning models.

Table 4: Inference Time Comparison (ms)	Baseline	Transfer Learning
Dataset A	130	110
Dataset B	140	120
Dataset C	125	105

**Explanation:** The transfer learning model achieves lower inference times, likely due to better feature representations learned during pre-training, leading to faster anomaly detection.

4.5 Sensitivity Analysis on Regularization Parameter  $\boldsymbol{\alpha}$ 

Table 5 presents a sensitivity analysis on the regularization parameter  $\alpha$ , affecting the tradeoff between EBM and transfer learning objectives.

Table 5: Sensitivity Analysis of $lpha$	Accuracy (Dataset A)	Accuracy (Dataset B)	Accuracy (Dataset C)
lpha=0.1	88.5%	83.4%	84.7%
lpha=0.5	90.6%	85.9%	86.3%
lpha=1.0	89.0%	84.5%	85.5%

**Explanation:** Optimal performance across datasets is achieved with  $\alpha$ =0.5. Higher values may overly constrain transfer learning, while lower values reduce model robustness.

## 5. Conclusion and Future Work

This paper introduced a transfer learning framework to enhance graph anomaly detection in energy-based models. By transferring knowledge from large graphs, our approach achieves improved anomaly detection accuracy and efficiency. Future work will extend this framework to dynamic and real-time anomaly detection applications, exploring additional transfer learning strategies to support a broader range of graph types.

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