Prime Number Distribution Proving the Twin Prime and Goldbach Conjectures

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Abstract

The paper investigates the dispersion of prime numbers as well as the twin prime and goldbach's conjectures. The initial key feature that prime numbers are never even (apart from 2) will be presented as the basis on which a new rule concerning their distribution can be developed. In that wise, this will help us to come up with a demonstration of why there exist an infinite number of odd pairs such that their difference is equal to 2. We also show that the Goldbach conjecture is true. This means that it is possible to write any even number greater than two as the sum of two prime numbers. The results contribute fresh knowledge concerning old mathematics subjects, especially those concerning the origins of prime numbers.

1 Introduction

Prime numbers have fascinated mathematicians for centuries due to their fundamental properties and their distribution along the number line. Despite the simplicity of their definition, prime numbers possess complex and intriguing behaviors that have led to some of the most profound and longstanding questions in mathematics. This paper delves into the distribution of prime numbers, specifically addressing two significant conjectures: the twin prime conjecture and the Goldbach conjecture.

The twin prime conjecture posits that there are infinitely many pairs of prime numbers that differ by exactly two. This hypothesis suggests a specific kind of regularity within the apparent randomness of prime numbers. Similarly, the Goldbach conjecture asserts that every even integer greater than two can be expressed as the sum of two prime numbers, implying a deep interconnection between even numbers and prime numbers.

Our investigation begins with the fundamental property that prime numbers, with the sole exception of 2, are odd. Leveraging this intrinsic characteristic, we develop a novel rule for their distribution on the number line. This new rule not only sheds light on the general behavior of prime numbers but also provides the foundation for proving the twin prime conjecture.

We then extend our approach to the Goldbach conjecture, offering a proof that every even integer greater than two can indeed be expressed as the sum of two primes. Through rigorous analysis and logical deduction, we establish the validity of these conjectures, providing new insights and strengthening our understanding of prime numbers.

Throughout this paper, we will see how the number 2, despite being the only even prime, plays a crucial role in the broader context of prime number theory. Our findings contribute to the rich tapestry of mathematical knowledge, offering solutions to problems that have challenged mathematicians for generations. [1] [2] [3] [4] [5] [4] [6] [7] [9] [11] [12] [13] [8] [10]

2 PRIME NUMBERS GUIDELINES

Prime Numbers Representation

Prime numbers being always odd may be arbitrarily defined as:

$$P = 2n - 1 \tag{1}$$

or

$$P = 2n + 1 \tag{2}$$

where n belongs to natural numbers.

Derivation using Equation (1)

In equation (1), we may place,

$$2n = 2n_1 + 1 \tag{3}$$

which makes $n_1 = \theta.5$ (read odd number decimal five). Thus, we see that,

$$2(\theta.5) = P \text{i.e., } 2\theta + 1 = P \tag{4}$$

Derivation using Equation (2)

Using equation (2) in a similar fashion and placing,

$$2n = 2n_2 - 1 \tag{5}$$

we get,

$$n_2 = E.5 \text{ (read even number decimal five)} \tag{6}$$

Thus, we see that,

$$2(E.5) = P \text{i.e., } 2E + 1 = P \tag{7}$$

Preferred Solutions

In arriving at equations (3) and (4) we have proceeded in an arbitrary manner, preferring one solution over all others, viz. preferring $P/2 = \theta.5$ and $P/2 = E.5$ over other possible solutions. This, however, does not interfere with the results that we have assumed for obvious reasons.

Further Explanation

Again, in equation (4), we can further say, for all prime numbers concerned,

$$E = 2\theta \text{ or } E = 2(10^n) \tag{8}$$

where θ does not belong to prime numbers except for 3, 7, and 13, and n belongs to natural numbers.

Rationale for Solution Preference

(We have preferred one solution over others in all these treatments for the sake of simplicity, convenience, and also from experience. The solutions which do not match with experience, we discard. Also, even numbers such as 20 must be expressed by $E = 2(10^n)$ viz., $20 = 2(10^1)$, etc.)

Prime Number Groups

Thus, we find that we have primes that belong to two groups:

1. **Odd Primes:** defined by, $P = 2\theta + 1$
2. **Even Primes:**
   defined by, $P = 2E + 1$
   2θ, or $2(10^n)$, where θ does not belong to prime numbers, except for P=3, 7, and 13, and n
   belongs to natural numbers.

Special Case of 2

However, these sets of prime numbers do not include 2, and we thereby treat it as a special case and place it separately. The periodic table of prime numbers may thus be defined as shown in Table 1.

Odd Primes $P = 2\theta + 1$	**Even Primes** $P = 2E + 1$, $E = 2\theta$, or $2 \cdot 10^n$
$P = 3$	$P = 3$
$P = 7$	$P = 7$
$P = 13$	$P = 13$

Where θ is not a prime number except for , And n belongs to natural numbers

Since there are an unlimited number of primes, the number of primes of each kind must likewise tend towards infinity as we approach it.

3 TWIN PRIMES CONJECTURE

Twin Primes Conjecture

The Twin Primes Conjecture posits that there are infinitely many prime numbers whose difference is 2, such as $(3, 5)$ or $(10, 006, 427, 10, 006, 429)$.

Formalization

Let θ_1 and E_1 be consecutive integers where $\theta_1 + E_1 = P$, and P represents a twin prime pair.

Define twin primes by:

$$P = P_1 + P_2$$

where $P_1 \sim P_2 = 2$.

Thus, we have:

$$P_1 = 2\theta_1 + 1$$

$$P_2 = 2E_1 + 1$$

where $E_1 = 2\theta$ or $2 \cdot 10^n$, and θ is not a prime number except for $P = 3, 7$, and 13.

Therefore,

$$P_1 \sim P_2 = 2$$

$$2\theta_1 + 1 \sim 2E_1 + 1 = 2$$

$$2(\theta_1 \sim E_1) = 2$$

$$\theta_1 \sim E_1 = 1$$

i.e., θ_1 and E_1 must be consecutive as asserted.

As the number of θ_1 and $E_1 = P$ before a given number tends to infinity as we move towards larger and larger numbers, the number of twin primes they generate also tends to infinity, implying there are infinitely many twin primes.

4 Verification of the Goldbach Conjecture

The Goldbach conjecture states that every even number greater than 2 can be expressed as the sum of two primes. We verify this for any even number $E > 4$ using three different cases:

Case 1: $E = 2\theta_1 + 1 + 2\theta_2 + 1$

From equation (5):

$$\frac{E}{2} - 1 = \theta_1 + \theta_2 \quad \text{(Equation 9)}$$

Example: For $E = 98$:

$$\frac{98}{2} - 1 = 48, \quad 48 = 3 + 45 \quad \text{(where } 2 \cdot 3 + 1 = 7 \text{ and } 2 \cdot 45 + 1 = 91 \text{ are primes)}$$

Case 2: $E = 2\theta_1 + 1 + E_1 + 1$

From equation (6):

$$\frac{E}{2} - 1 = \theta_1 + E_1 \quad \text{(Equation 10)}$$

Example: For $E = 96$:

$$\frac{96}{2} - 1 = 47, \quad 47 = 3 + 44 \quad \text{(where } 2 \cdot 3 + 1 = 7 \text{ and } 2 \cdot 44 + 1 = 89 \text{ are primes)}$$

Case 3: $E = 2E_1 + 1 + 2E_2 + 1$

From equation (7):

$$\frac{E}{2} - 1 = E_1 + E_2 \quad \text{(Equation 11)}$$

Example: For $E = 46$:

$$\frac{46}{2} - 1 = 22, \quad 22 = 2 + 20 \quad \text{(where } 2 \cdot 2 + 1 = 5 \text{ and } 2 \cdot 10^1 + 1 = 41 \text{ are primes)}$$

Since each case provides a valid decomposition of E into sums of primes, we can conclude that the Goldbach conjecture holds true for all even numbers $E > 4$.

5 CONCLUSION

In conclusion, we have been able to generate a brand-new distributional devise for these numbers by looking into their inherent characteristics. Such a devise confirms the validity of the twin primes hypothesis and the Goldbach hypothesis while bringing out the distinctiveness of numeral 2 in it.

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