Estimation of Height of a Shape a 2D Image from Its Shadow Using Neural Networks

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Summary: Determining the height of objects in a 2D image from their shadow and developing a model capable of estimating the height of uniform geometric shapes and objects of different sizes from the shadow they cast, is the fundamental part of this research, which proposes the use of convolutional neural networks (CNN) as a Machine Learning (ML) technique for pattern detection, feature extraction from shadows and shapes in images. For this purpose, a dataset was constructed with photographic images of shapes or objects, as well as their shadows cast from different angles and locations. The structure of the proposed dataset is characterized by the name of the shape, the name of the shadow, the length of the shadow, the height of the shape and the angle of the light source, which together allow to improve the accuracy of the model. In this sense, the research focuses on the analysis of lights and shadows of different geometric shapes or objects within a 2D image, where the projected shadow is the information to be used and with which it is intended to determine the height of the shapes or objects. The main reason for this research is oriented towards people who have visual impairment either total or partial, and that from the touch pretend to define or differentiate an object, becoming impossible if asked to indicate which of these are present in a 2D image (photograph or painting). Therefore, it is essential that from a 2D image is to highlight the objects to be represented in a 2.5 model, where the height of the selected objects will be the key to create the model. In this sense the project to be developed seeks to determine the height of predefined objects in a two-dimensional image from its shadow.

Keywords: Shadows, Shapes, Convolutional neural network, Dataset, 2D, Height.

1. Introduction

This Estimating the height of shapes or objects in a 2D image from their shadows is a challenging process when they are complex and irregular under uneven lighting conditions. This has presented its own challenges, such as the need for an accurate model that estimates the geometry of the object and the way light strikes it. Thus, the height of an object can provide relevant information about its shape, size and position. However, estimating the height of an object or shape in a 2D image from its shadow is a more complex problem, due to the lack of three-dimensional information about it. In this context, the use of neural networks has become a valuable tool to address this problem. In this research, a neural network-based solution for height estimation from its shadow is presented. This model uses a convolutional neural network architecture to extract relevant features from the image to perform height estimation.

To train and evaluate the model, a dataset was developed with photographs of different geometric shapes indicating a variety of shapes and shadows under different lighting conditions. The proposed model can accurately estimate the height of shapes under certain conditions, suggesting that it could offer a promising solution for estimating the height of shapes in 2D images from their shadow, which could have important applications in various fields, having the potential to significantly improve machine perception capabilities in various applications. The work will focus on evaluating the accuracy and efficiency of the proposed method, and comparing it with other existing algorithms for estimating the height of objects from their shadow.

The network is trained on a labeled dataset consisting of 2D images, their corresponding heights and their shadows, using the error backpropagation algorithm to minimize the difference between the estimated heights and the actual object heights. The approach proposed here was evaluated on a test data set and significant accuracy in estimating object heights was obtained compared to other existing methods. In addition, a sensitivity analysis was performed to evaluate the influence of different model parameters on the accuracy of height estimation. This approach is especially useful in situations where the image quality is low or there are ambiguities in the shape of objects. That is, the integration of cast shadow information into a learning model can be a promising approach, to solve the problem of estimating the height of a shape from its shadow in a 2D image.

By using these deep learning techniques, such as neural networks, large amounts of data can be processed and analyzed, allowing for greater accuracy in estimating the height of a shape from its shadow. In addition, the ability to integrate new geometric shapes and shadows within the set of images used in the learning model makes this technique highly adaptable and scalable for future research in which it is implemented with irregular shapes.

Similarly, in [9] (S. Mohajerani, P. Saeedi. 2018) addressed their research on automatic detection in an image of shadows using a segmentation method and the use of Deep Learning, where what they do is to identify the regions of shadows at the pixel level in an RGB image, whereby they manage to extract the features of the shadows, providing a Convolutional Neural Network (CNN) with knowledge to detect patterns of the shadows locally and globally and prior information of the illumination source and the dynamics of the objects in the image.

On the other hand [25] (Tao, M. W., Srinivasan, P. P., Hadap, et al. 2017) proposes an algorithm based on dense depth estimation, which combines blur and matching metrics. Furthermore, it defines an optimization framework that integrates photo coherence, depth coherence and shading coherence, for light field depth estimation in different scenarios. This algorithm incorporates the blur, correspondence and shading signals outperforming other more advanced algorithms.

1.1. The Shadow

Shadows can be defined as parts of the scene that are not directly illuminated by a light source due to an obstructing object or objects. A similar definition is posited in shadow detection and they likewise posit a typical shadow that could be divided into two different types. One type is denoted as self shadow where the shadow region is on the object itself. The other type is the cast shadow for which the shadow region is on the background or on other objects. Here the cast shadow is usually divided into two parts, umbra and penumbra (Fig. 1).

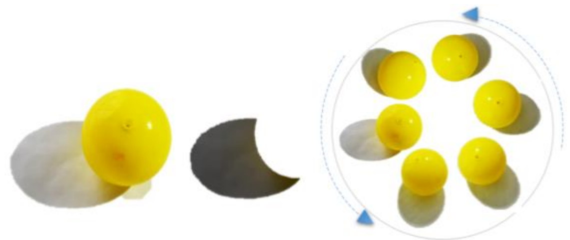

Fig. 1. Figure and corresponding shadow and Rotation of the light source.

In such a way that in [4] (Vicente, Samaras. 2014) in 2D digital images, different scenes can be found, which contain elements such as textures, edges, shapes, colors, shadows, etc., and shadows from which relevant information can be extracted and used in investigations by implementing shadow detection algorithms. This information is used to establish the relationship between the geometry of the object, the light source and the shadow area (Vicente, Samaras 2014).

Similarly, in [8], [6], (Kriegman, Belhumeur. 1998) and [11] (D. C. Knill, et al. 1997) it is also understood that shadows are a source of relevant information at the level of shapes of surfaces or objects, allowing to locate areas of interest in an image, direction of the illumination source, geometry of the shape, among other characteristics. Likewise, in [11] (Knill, Mamassian and Kersten. 1997) and in [17] (S. A. Shafer, T. Kanade. 1983), their research indicates that these geometric properties of shadows are of special interest, because they allow establishing perceivable relationships between shapes, shadows and illumination, according to the structure and height of the surface.

In many previous works in [24] (Hintze, Morse. 2019), the information that is extracted from the shadow, by means of techniques, methods and algorithms with purposes such as: determining the location of the shape or object, area of interest, direction of the illumination source among others, is usually oriented to its later elimination, where this makes its transcendence not so remarkable.

In [12] (Salvador, Cavallaro and Ebrahimi. 2004) shadows in a 2D image have relevant information about the scene, the location of the shape or object, the characteristics of the surface and the light source are obtained. He mentions techniques that are based on models that represent knowledge of the geometry of the scene, the objects or shapes, the light source, and techniques based on properties that identify shadows by using features such as geometry, brightness and color of the shadows. (See Fig. 2).

This provides several important elements where the shadow is one of the essential parts of the research, because it allows inferring shapes of objects, and when present in an image provides information that can be used to determine the shapes and orientations of objects.

Similarly, in [12] (Salvador, Cavallaro and Ebrahimi. 2004), address general aspects related to digital image processing, using convolutional neural networks (CNN's) with different data sets (datasets), where the latter have as main feature, images of geometric primitives and the shadows they cast according to an illumination source in a specific location. Similarly, in [2] (Panagopoulos, Hadap, Samaras and Dimitris. 2013) in their research employ synthetic images divided into small rectangular regions, and therefore, for each of these it is possible to capture what corresponds to the distribution of intensities and the ability to handle surfaces. At this point the geometry sections are combined, the intensity distribution is integrated into a dictionary that with all the sections can generate a hypothesis about that shape.

Using the learning model that processes the images of the training set, which with all the sections or small regions form a large dictionary, and where the decomposition of an image in shadow and reflectance images describe the perception of brightness for these images.

In [1], [13] (Hosseinzadeh, S., Shakeri, M. and Zhang, H. 2005) it is shown that the concept of sections corresponds to a collage of sections with a different reflectance and which are illuminated with an illumination source that varies slowly in intensity, and with shapes that are of a constant scale factor. This learning model with the data set to be generated would be used to determine the surface height and 3D generation of the object or shape, initially taking the information of the shadow it casts on a 2D image. In [12] (Salvador, Cavallaro, Ebrahimi, Touradj. 2004), [7] (G. Liasis, S. Stavrou. 2016) illumination, shadows and reflectance are an integral part of the different scenes, providing relevant information about the shape and appearance of the object.

In [2] (Panagopoulos, Hadap, and Samaras. 2013) propose the use of a dictionary use of a dictionary (Dictionary) that is conformed by different patches (Patches) of geometry associated to the image, to a distribution of pixel intensities that can generate that geometry and with the use of larger image regions that allow capturing appearance data, it is possible with a data set and supported in neural networks it is possible to design a model that fits the proposed research.

In [22] (Varol, Shaji, Salzmann, Fua. 2011) has an approach based on deformable shape recovery that can work under complex illumination and on partially textured surfaces. They used an algorithm that performs a learned mapping of the intensity distribution and shape of local surface patches, which is focused on 3D surface shape estimation.

In [14] (D. S. Kim, M. Arsalan, K. R. Park. 2018) they employ a Convolutional Neural Network (CNN) for shadow detection using cameras and using the open database CAVIAR (Context-Aware Vision Using Image-Based Active Recognition). They indicate that in their research it is not computationally expensive both in training and validation and can be adapted to tasks such as image segmentation and has a level of accuracy of other methods.

1.2. The Dataset

The experimentation process is carried out under controlled conditions in terms of illumination, shapes and surface. Such conditions were used in the model allowing initially to test the model of the network and that it learned certain characteristics, which is why it was decided to use the different photographs of three geometric shapes on a uniform surface, with a dimmable LED lamp and the shadows corresponding to each shape (Fig. 3).

It is important to mention that the data set was oriented with the objective of capturing photographs under different lighting conditions to give an initial knowledge to the neural network.

The data set includes 800x600 color photographs (RBG) of various geometric shapes of various colors, taken from a distance of 90 cm from the edge of the surface. The surface where the shape or object is located is white, smooth and does not reflect light and is made of white matte adhesive vinyl. The surface has a size of 90 cm and a diameter of 180 cm, was designed circular to facilitate the displacement of the LED lamp, is made of MDF (Medium Density Board) wood of 9 mm (Fig. 4).

Figure / Units	Measurement units. 1 unit = 1 cm																	
	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	7	8	9	9.5	10	10.5	Total
Cube	✓	✓	✓		✓		✓	✓	✓	✓	✓	✓	✓	✓		✓		1.500
Cylinder	✓	✓	✓		✓		✓	✓	✓		✓	✓	✓	✓		✓	✓	1.500
Sphere	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			✓		✓			1.500

Total number of images in the dataset: 4.500

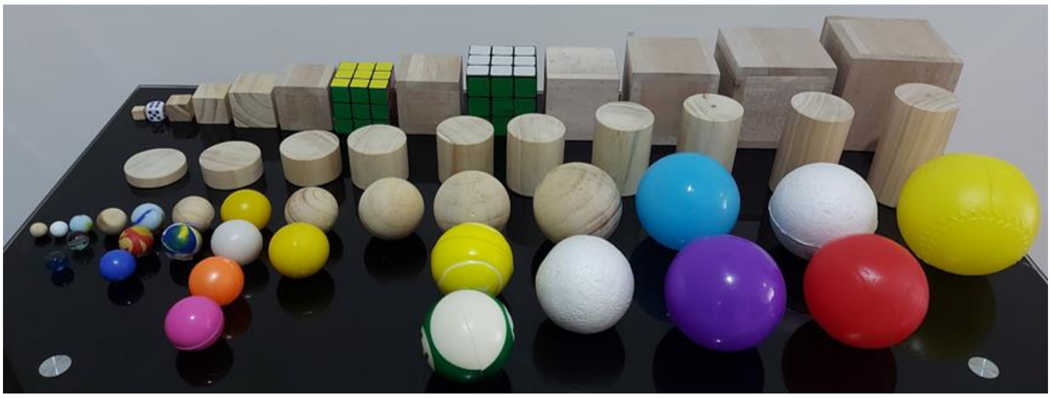

Fig. 3. Characteristics of the Dataset.

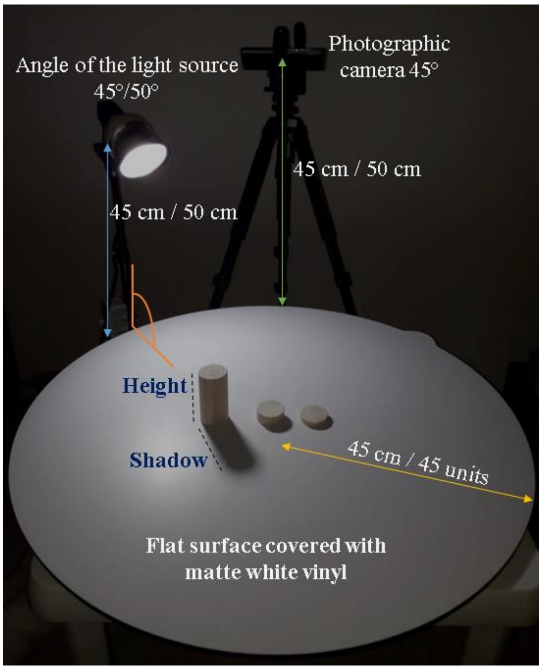

Fig. 4. Mock-up for taking photographs.

1.3. Model Description

With the research conducted, it is proposed to use and/or design a convolutional neural network, which through the training of a set of data will provide the necessary knowledge to predict the height of an object or shape with the projected shadow. With the learning process, it is expected that the network, by using as input information the shadow cast by the shape or object in a 2D image, will predict or determine the height of the object in question. The data set (Dataset) was designed and developed with objects and/or basic geometric shapes (sphere, cube, cylinder) to perform the experimentation in a controlled manner, since after this proposal and as a future work it is expected to increase the data set with more complex shapes and shadows. The total amount of images in the dataset is approximately 4500 color photographs and for each shape there are 1500 photographs. (See Table 1. Height Distribution of Geometric Shapes).

It is important to note that the proposed model (Figs. 5, Fig. 6) uses geometry of three-dimensional shapes of the objects or shapes with the corresponding shadow from different locations in the plane and generated by a single light source. The latter is rotated around the figure to obtain shadows from different angles. The light source is located 90 centimeters from the center of the object to the base of the lamp and the heights of the lamp vary from 45 centimeters (cm) to 50 centimeters (cm) so that the different shadows are observable on the surface under varying intensities of illumination (See Fig. 1, Rotation of the light source).

Model: "sequential"

Layer (type)	Output Shape	Param #
conv2d (Conv2D)	(None, 98, 98, 16)	448
max_pooling2d (MaxPooling2D)	(None, 49, 49, 16)	0
conv2d_1 (Conv2D)	(None, 47, 47, 32)	4640
max_pooling2d_1 (MaxPooling2	(None, 23, 23, 32)	0
conv2d_2 (Conv2D)	(None, 21, 21, 64)	18496
max_pooling2d_2 (MaxPooling2	(None, 10, 10, 64)	0
flatten (Flatten)	(None, 6400)	0
dense (Dense)	(None, 512)	3277312
dense_1 (Dense)	(None, 1)	513

Total params: 3,301,409
Trainable params: 3,301,409

Fig. 5. Model Architecture.

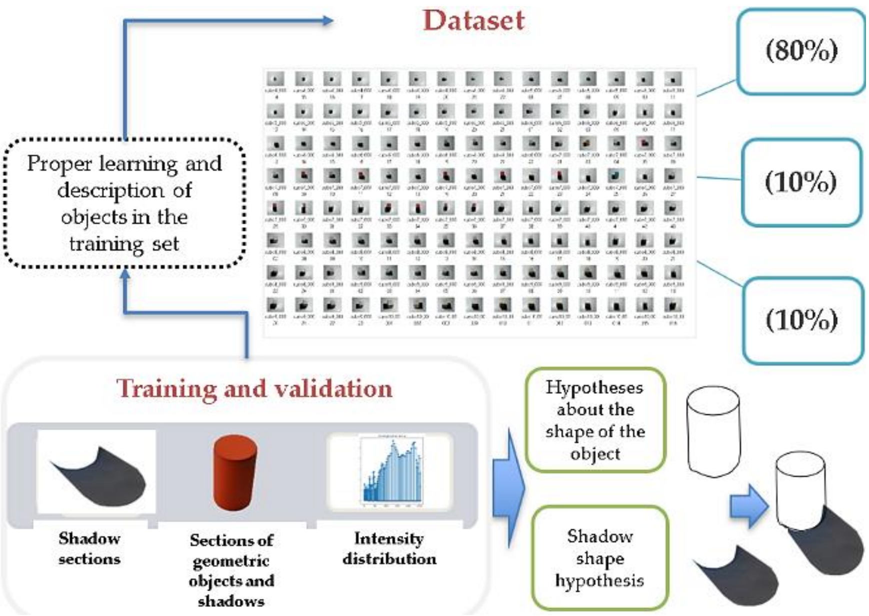

Fig. 6. General Scheme of the Proposed Model.

Table 1. Height Distribution of Geometric Shapes.

Figure	Color	Measurement (cm)	Quantity
Cube	Brown	I	1
Cube	Brown	1,5	1
Cube	Brown	2	1
Cube	Brown	3	1
Cube	Brown	4	1
Cube	Brown	5	1
Cube	Rubik's Cube	5,5	1
Cube	Brown	6	1
Cube	Rubik's Cube	6,5	1
Cube	Brown	7	1
Cube	Brown	8	1
Cube	Brown	9	1
Cube	Brown	10	1
Total Cubes			**13**
Cylinder	Brown	1	1
Cylinder	Brown	2	1
Cylinder	Brown	3	1
Cylinder	Brown	4	1
Cylinder	Brown	5	1
Cylinder	Brown	6	1
Cylinder	Brown	7	1
Cylinder	Brown	8	1
Cylinder	Brown	9	1
Cylinder	Brown	10	1
Cylinder	Brown	10,5	1
Total Cylinder			**11**
Sphere	Brown	1	1
Sphere	White	1,5	1
Sphere	White	2	3
Sphere	Brown	2,5	1
Sphere	White, Yellow, Blue	3	3
Sphere	Brown, Blue with yellow	3,5	2
Sphere	Yellow, White, Orange, Pink	4	4
Sphere	Brown, Yellow	4,5	2
Sphere	Brown	5	1
Sphere	Brown, Yellow, Green,White	6	3
Sphere	Brown, White	7	2
Sphere	Blue, Violet	8	2
Sphere	White, Red	8,5	2
Sphere	Yellow	9	1
Total Sphere			**28**

With this knowledge it is expected that the convolutional neural network will be able to predict the height of the shapes from their shadow. This height measurement is given in centimeters (units) using as input image only the projected shadow. Thus, it is initially expected to effectively estimate the height of the shape or object corresponding to the shadow, which is one of the challenges of the research. Moreover, there is additional information in the dataset that combined with appropriate 2D to 3D shape reconstruction algorithms it is possible to obtain the reconstruction of the shape according to the height established from the shadow.

In [21] (M. Daum, G. Dudek. 1998) the reconstruction of 3D surfaces from shadows is reported. Here, the movement of shadows that are cast is used to obtain information about the scene structure, based on the collection of images from a fixed point as an illumination source moves. As a result, a 3D scene reconstruction algorithm is obtained taking into account the trajectories of the light source.

On the other hand, in [26] (Huang, X., Gao, J., et al. 2007) their research addresses shape estimation from shading (SFS) which aim to solve a problem with few constraints to estimate the depth map from a single image, developing an example-based method to improve the accuracy of (SFS), improving the reconstruction quality from real images of different shapes by obtaining prior knowledge of their appearance for three-dimensional shape recognition.

Similarly in the proposal in [20] (E. Prados, O. Faugeras. 2006) with the shape from shading (SFS) and with a single image they estimate the 3D shape of a surface, where in diffuse surface the position of the illumination source is not known and where the reflectance map is not known.

In [15] (S. Savarese, et al. 2007), [16] (S. Savarese, et al. 2002) and [5] (R. Gouiaa, J. Meunier. 2014) they propose a new method to recover a shape from shadow (SFS). These projected shadows are relevant information about the shape of objects, discovering cavities that are not available from clues such as occluded boundaries. This method is called "occluding boundaries". According to the volume occupied by an object it is possible to identify and sculpt the regions of the volume that present inconsistencies with the pattern observed in the shadows. In summary, their proposal is a reconstruction system to recover the shape from silhouettes and shadow carving where shadow carving is used to carve the concavities and silhouettes are used to reconstruct the initial conservative estimate of the shape of the object.

On the other hand, in [3] (D. Forsyth, A. Zisserman. 1991), he shows how mutual illumination and image irradiance within an image give rise to more complex structures.

Similarly, in [27] (Atkinson, G. A., & Hancock, E. R. 2007), research presents a novel method for the reconstruction of 3D surfaces with polarization and shading information from two views. They use Fresnel theory in image processing to obtain estimates of the surface normals. Here they emphasize how the measured pixel brightnesses depend on the surface orientation combined with the refined estimates to determine the correspondence between two views of an object. In [19] (R. Liu, S. Menon, et al. 2022) they address 3D reconstruction when the object structure is partially or totally occluded. They introduce into the method projected shadows of an unobserved object, to perform the inference of the possible 3D volume to which it corresponds. The "inheritable" imaging model allows to jointly infer both the 3D shape of the object, its location and that of the illumination source.

Likewise, in [23] (D. Samaras, D. Metaxas. 2003) they present a method for the integration of nonlinear holonomic constraints in deformable models and its application to the problems of shape and direction estimation of the illumination source from hats. Their proposal indicates that it works when the direction of the light source is not known. They coupled the shape estimation method with a light estimation method where better shape estimation results in better light estimation and vice versa.

In [19] (R. Liu, S. Menon, et al. 2022) they address 3D reconstruction when the object structure is partially or totally occluded. They introduce into the method projected shadows of an unobserved object, to perform the inference of the possible 3D volume to which it corresponds. The "inheritable" imaging model allows to jointly infer both the 3D shape of the object, its location and that of the illumination source. The advantage of this approach is the estimation of multiple 3D scenes from shadows.

The data set is organized in folders and within each subfolder there are images corresponding to each geometric shape, the shadow, the height. It is important to point out that with the experimentation process, the length of the shadow and the angle of the light source were added to the Dataset already created, improving the accuracy of the model. (See Fig. 8. Dataset Folder Structure).

Some research such as in [10] (Sanin, C. Sanderson, B. C. Lovell. 2012) presents various methods and algorithms for detection, removal of moving shadows as of 2012, and places them in a feature-based taxonomy that is made up of four categories, chromaticity, physics, geometry, and textures. The use of tracking performance is proposed as an approach to determine shadow detection methods and algorithms. Of the most prominent methods are the method based on chromaticity is the fastest to implement and execute, but sensitive to noise and fails when the spectral properties of objects when these are similar to those of the background and the method based on textures of large and small regions where the results are more accurate, although its computational cost is high.

Likewise, in [18] (S. H. Khan, M. Bennamoun et al. 2014) they use multiple deep convolutional neural networks (ConvNets), where their architecture consists of alternating convolution and subsampling layers. This research is focused on shadow detection, having as input a database containing in various lighting conditions with sunny, cloudy and dark environments. For this they used a Conditional Random Field (CRF) model, which translates into predictions for each pixel of each test image, and are subsequently compared with the actual shadow masks. This enforces labeling consistency across the nodes of a gridded graph defined on the image by eliminating isolated labeling results.

3. Distribution and Structure of the Dataset

For the organization of the dataset, these were grouped according to the height measurements and their geometric shape. The dataset is made up of geometric figures such as spheres, cubes and cylinders of different heights.

The measurements were taken in centimeters, which are then handled in units, in order to facilitate the reuse of the dataset in any other measurement system. (See Fig. 7).

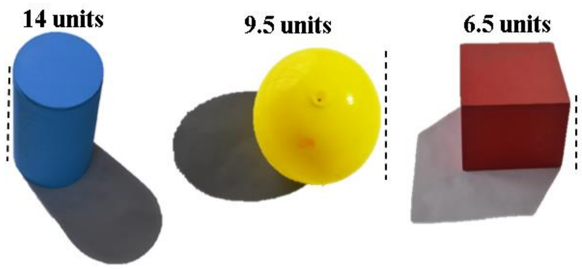

Fig. 7. Height Cylinder, Sphere and Cube.

Initially two variables (features) were defined in the dataset, the first being the type of figure or shape and the second the height of the 3D object. Subsequently, three more variables are added to the existing ones, these are the shadow, the shadow length and the angle of the light source. Therefore, a total of five more variables are included and implemented in the training, validation and test set. These variables are within the model and were used in the learning process to improve the accuracy of the model.

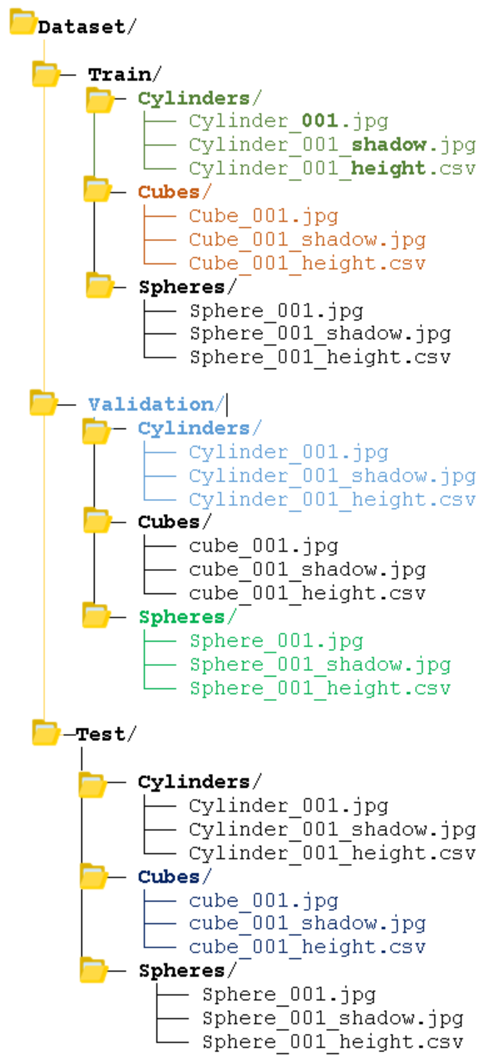

Fig. 8. Dataset Folder Structure.

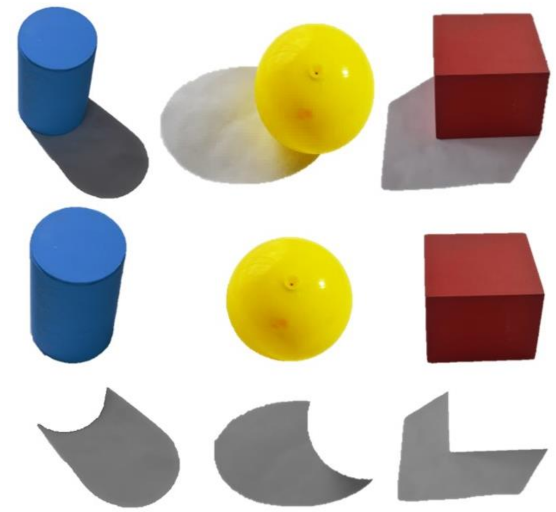

Fig. 2. Types of dataset figures

At the time of performing the network learning process, it is taken into account that it is distributed or divided into a training set (to build the model), a validation set (to test different parameters of the learning algorithm) and a test set (to evaluate the quality of the model). A suggested distribution of the data could be as follows, 80 % for the training set, 10 % for the validation set and 10 % for the test set. (Tables 2 and 3). Similarly, it could also be 70 %, 15 % and 15 % respectively.

Table 2. Dataset Distribution 1.

Dataset	Figures	Training	Validation	Test
Dataset No.1	Spheres	661	47	47
	Cylinders	195	47	47
	Cubes	269	47	47
Subtotal		**1.125**	**141**	**141**
Total images (100%)		**1.407**		
Dataset distribution		**80%**	**10%**	**10%**

Table 3. Dataset Distribution 2.

Dataset	Figures	Training	Validation	Test
Dataset No.2	Spheres	1200	150	150
	Cylinders	1200	150	150
	Cubes	1200	150	150
Subtotal		**3.600**	**450**	**450**
Total images (100%)		**4.500**		
Dataset distribution		**80%**	**10%**	**10%**

4. Conclusions

Regarding the scope and challenges of this research, one can indicate that related to the interpretation of 2D images by visually impaired people, since they cannot distinguish the objects in them from touch. However, 3D objects can be useful to provide a more complete understanding of the image. This aims to contribute to improve the accessibility and inclusion of visually impaired people in the interpretation of images and their environment. In this context, the present project aims to determine the height of predefined objects in a 2D image from their shadow. This method can be especially useful, allowing them to understand the visual information in an image. To achieve this goal, Machine Learning techniques are used, specifically convolutional neural networks (CNNs) that allow to detect relevant patterns and features in images. (See Fig. 5). Therefore, a dataset will be created to obtain images of objects and their shadows, which will be used to train, validate and test the convolutional neural network (CNN).

Experimental results show that the proposed method achieves significant accuracy in estimating the height of shapes from their shadows compared to other traditional computer vision methods. Additionally, the construction of the dataset is also an important contribution, since there are not many Datasets available that meet the specific requirements of this project. All in all, it is hoped that this research can be useful for future work.

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