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Towards Efficient Solvers for Optimisation Problems

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***Abstract*—*Constraint programming* is pervasive and widely used to solve real-time problems which input data could be scaled up to the huge sizes, and the results are required to be given efficiently and dynamically. Many technologies such as *constraint programming*, *hybrid technologies*, *mixed integer programming*, *constraint-based local search*, *boolean satisfiability* could have different solvers and backends to solve the real-time problems. *Streaming videos* problem is the problem that requires to decide which videos to put in which cache servers in order to minimise the waiting time for all requests with a description of cache servers, network endpoints and videos are given. In this paper, we will model the *streaming videos* problem in two different ways. The first model will be implemented using heuristics, and the *global constraints* will be used in the second model. The experiments will be benchmarked using *MiniZinc*, which is an open-source constraint modelling language that can be used to model constraint satisfaction and optimisation problems in high-level, solver-independent way. The aim of the paper is to benchmark those technologies to evaluate the execution time and final scores of the two models using large instances of input data from Google Hash Code.**

***Index Terms*—optimsation, constraint programming, modelling**

I. Introduction

Nowadays, watching videos online is pervasive, especially watching videos from Youtube. When streaming videos from Youtube to a huge amount of people, who could be in the same city or from different continents, minimising the waiting time for all requests from clients are critical. In the context of the *Streaming videos* problem, the video-serving infrastructure includes *remote data centers* locating in thousands of kilometers away, *cache servers* which store copies of popular videos, and *endpoints* which each of them represents a group of users connecting to the Internet in the same geographical area. The expected solution for the *Streaming videos* problem is to decide which videos to put in which cache servers. The specification of the problem could be found in detailed at [1], and the data could be found at [2]. *MiniZinc* [3] is a constraint-based modelling language for satisfaction and optimisation problems such as Streaming videos problem with independent solving technologies which supports for diverse technologies' solvers for instances constraint programming (CP), constraint-based local search (CBLS) [4], mixed-integer programming (MIP), boolean satisfiability (SAT), SAT modulo theories (SMT), and hybrids, such as CP with lazy clause generation (LCG) [5]. In this paper, the *bin-packing* approach, which is modelled in modelling language *MiniZinc*, will be used to solve the *Streaming videos* problem in two different ways: use the built-in global constraint `bin_packing_load`, and model the problem using heuristic.

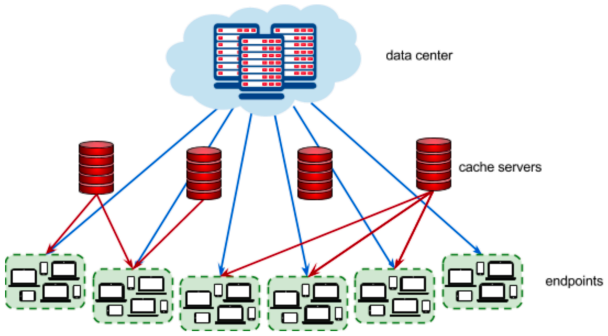

Figure 1: The video serving network

II. Backgrounds

Given a description of cache servers, network endpoints and videos, along with predicted requests for individual videos, the task is to decide which videos to put in which cache servers in order to *minimise* the average waiting time for all requests. In other word, the task is to *maximise* the average *saving time* for all given requests. Figure 1 illustrates the video serving network, which includes the data center, cache servers, and endpoints [1]. The **data center** stores *all videos*. The sizes of videos, the maximum capacity of cache servers are in megabytes (MB). Each *video* can be put in 0, 1, or more cache servers. Each *cache server* has a maximum capacity. Every *endpoint* is connected to the data center, however, it may be connected to 0, 1 or more cache servers. Each endpoint is characterised by the latency of its connection to the data center, and by the latencies to each cache server that it is connected to. The *predicted requests* provide data on how many times a particular video is requested from a particular endpoint.

Table I illustrates the input file [1]. The original input data is given in the format that is not the instance for *MiniZinc*. Consequently, pre-processing the original inputs to *MiniZinc*'s instances is taken in the first place. The conversion of the output instances which are compatible with *MiniZinc* will be done by Python script together with precomputations [6],

[7]. Table II illustrates the output file. Notice that the output example is not optimal solution. A better solution is to put videos 1 and 3 in cache 0 to maximise the saving time since the latency of cache 0 is minimal.

5 2 4 3 100	5 videos, 2 endpoints, 4 request descriptions, 3 caches 100MB each.
50 50 80 30 110	Videos 0, 1, 2, 3, 4 have sizes 50MB, 50MB, 80MB, 30MB, 110MB.
1000 3	Endpoint 0 has 1000ms datacenter latency and is connected to 3 caches:
0 100	The latency (of endpoint 0) to cache 0 is 100ms.
2 200	The latency (of endpoint 0) to cache 2 is 200ms.
1 300	The latency (of endpoint 0) to cache 1 is 300ms.
500 0	Endpoint 1 has 500ms datacenter latency and is not connected to a cache.
3 0 1500	1500 requests for video 3 coming from endpoint 0.
0 1 1000	1000 requests for video 0 coming from endpoint 1.
4 0 500	500 requests for video 4 coming from endpoint 0.
1 0 1000	1000 requests for video 1 coming from endpoint 0.

Table I: Example of input file.

3	We are using all 3 cache servers.
0 2	Cache server 0 contains only video 2.
1 3 1	Cache server 1 contains videos 3 and 1.
2 0 1	Cache server 2 contains videos 0 and 1.

Table II: Example of output file.

The paper makes the following contributions: microbenchmarks that compare the CP, LCG, MIP, CBLS, and SAT's `bin_packing_load` global constraint versus manual model.

III. Models

a) Manual model: In the model, two 2D-matrix arrays `usedCache` and `vInDc` are defined. The first 2D-matrix array `usedCache[..,..]` represents decision variables of which videos will be put in which corresponding cache. The domain value of each element of `usedCache` is $\{0, 1\}$.

```
% ci : id of cache server being described
%      ( 0 .. C : 1.000 ), 0 : data center
% vi : id of videos stored in this cache server
%      ( 0 .. V : 10.000 )
array[CACHE, VID] of var {0,1} : usedCache;
```

In order to mark which videos are put in the data center because their sizes exceed the capacity of caches connecting to corresponding endpoint, another 2D-matrix array `vInDc[..,..]` is defined.

```
% ci : id of cache server being described
%      ( 0 .. C : 1.000 ), 0 : data center
% vi : id of videos stored in this cache server
%      ( 0 .. V : 10.000 )
array[CACHE, VID] of var {0,1} : usedCache;
```

There are 6 constraints, 3 functions, and 2 precomputations are introduced into this model. The decision variable `score` is bound to the `savingTime` to avoid the division / and *div*.

```
% Decision variable: total time that saved for all requests
%     nReq is parameter, the division and multiplication
%     could be performed at the output phase.
% score = (savingTime / nReq ) * 1000
var int: score;
```

The final score is computed in the output phase by dividing `savingTime` by total requests `nReq`, and then multiplying by 1000.

```
%       ["\nscore: \((score/nReq)*1000)"
```

The first precomputation (line 65–68) calculates the total number of used caches in the 2D-matrix `usedCache`.

While the second precomputation (line 102–103) iterates over all the requests and gives the total number of all requests.

Three functions are defined in this model. The first function `selectedVideo()` (line 105–109) takes two parameters, which are cache `ca` and video `rv`, and checks whether the video `rv` already stored in any other caches. It returns 0 if the video is not stored in caches other than cache `ca`. The second function `hungryCache` (line 111–116) takes two parameters, cache `ca` and video `vi`. Before storing the new video `vi` into the cache `ca`, the spare capacity of the cache is checked to make sure that the total capacity does not exceed the given maximum capacity of the cache. The last function `emptyCache` (line 118–120) takes one parameter cache `ca` and check whether the given cache `ca` is empty or not.

The constraint C2 (line 79–83) guarantees that the total sizes of all stored videos in a cache does not exceed its maximum given capacity. The constraint C3 (line 86–99) computes the total number saving time of all caches and all requests. With all the empty cache, the unrequested videos will be stored in the cache. The constraint C4 (line 122–128). It iterates over the endpoint `ENDPONIT`, cache `CACHE`, and video `VID`. In this constraint, the unrequested video `vi` is stored in the cache `ca` under the following conditions: (1) there is connection between endpoint and cache `eConCache[ en, ca ] > 0`, (2) the considered video could be possible to store in the cache `vInDc[ en, vi ] = 0`, (3) the considered cache is empty `emptyCache(ca)`, (4) the cache doesn't exceed its limit when storing the video `hungryCache(ca, vi)`. The requested videos will be stored in the cache in constraint C5 (line 131–136) by using function `selectedVideo` to check whether there is connection between cache and endpoint `eConCache[ en, ca ] > 0`, the video `vi` is not stored in any other caches `selectedVideo(ca, vi) = 0`, and the size of videos does not exceed the capacity of the cache `vInDc[ en, vi ] = 0`. To avoid the duplication of stored videos, the constraint C6 (line 138–142) is defined. To all caches connecting to the endpoint, the `at_most()` restricts that each requested video could be stored only in one of those connected caches `at_most(1, [ usedCache[ ca, vi] | ca in CACHE ], 1)`.

Redundant decision variables `vInDc` are introduced into the model to mark which videos are stored in the data center. The `vInDc` reduces the search space when iterating over nested loops such as `VID`, `ENDPOINT`, and `CACHE`. The reified constraint in the model gives the solution of which videos are stored in the data center. When the size of a video exceeds the capacity of a cache or an endpoint does not have any connected cache server to store requested videos.

```
constraint forall(req in REQUEST)(
  let { int: rv = request[req, Rv];
        int: re = request[req, Re];
  } in ((videoSize[rv] > x \/ endpoint[re, K] = 0)
  <-> vInDc[re, rv] = 1 )
);
```

The 2D-matrix `usedCache[ CACHE, VID ]`, which represents the final result in the streaming videos problem, does *not* introduce the symmetries. Since each cache has different latency, swapping the cache rows in the `usedCache` might produce a non-optimal result. Similarly, swapping any number of columns which is corresponding to the stored

videos in the usedCache solution might leads to a non-optimal result also.

b) Global constraint model: The bin_packing_load constraint could be used as an alternative model. The *bin_packing_load(array[int] of var int: load, array[int] of var int: bin, array[int] of int: w)* constraint requires that each item i with weight w[i] be put into bin bin[i] such that the sum of the weights of items in each bin b is equal to load[b]. In this problem, with the view point of video serving network, capacity load[i] must be no greater than given capacity X of each cache server. The weights of each item, w[i], corresponds to the videos size reqVid[i]. Each *cache server* is corresponding to one *bin*, so C cache servers corresponds to C bins. While the videos that are not requested or exceed the capacity of cache servers will be stored in the data center.

The *bin_packing_load* model includes constraints that consider the *caches* as *bins*, with maximum capacity and loading capacity. The videos that are stored in data center are implicitly captured by parameter reqVid.

In the alternative *bin_packing_load* model for Streaming Videos problem, the ::int_search annotation is used to compute the final *score* with array variables, which concatenate the bin array and load array. The next argument first_fail specifies that the variables are chosen in the order that appear. To those chosen variables, the assignment annotation indomain_min will assign the largest video size in the bin and load domain. Ultimately, the strategy annotation complete is specified.

```
99  solve :: int_search(
100         bin ++
101         load
102         ,
103         first_fail, indomain_min, complete) maximize savingTime;
```

IV. Experiments

All experiments were run under Linux Ubuntu 16.04 (64 bit) on an Intel Xeon E5520 of 2.27 GHz, with 4 processors of 4 cores each, with a 24 GB RAM and an 8 MB L2 cache (a ThinLinc computer of the IT department). The two models could be found at [1]. We have chosen the backends for Gecode, Chuffed, Gurobi, OscaR.cbls, and Lingeling. Table III gives the results for various instances IV on the Streaming Videos model. The time-out was 600000 milliseconds.

The experiment is done using two different version of *MiniZinc*, 2.1.7 and 2.2.1 as it is recently released. In the first experiment, all the instances are conducted using *MiniZinc* 2.1.7. The test results produced by *MiniZinc* 2.1.7, and *MiniZinc* 2.2.1 are marked by (*) and ($^\psi$), respectively. In order to run the test in all backends, the final score computation is done at the output phase to avoid the division computations such as / and *div* which are not executable in *Chuffed* and *Gecode*. Ultimately, the significant difference between two version is the execution time which is illustrated in Figure 2. Overall assessment, *MiniZinc* 2.1.7 produces the result in the shorter time than the latest version 2.1.7. For instance, running *warm_up* instance, *MiniZinc* 2.2.1 produces the result in 0.457 second while *MiniZinc* 2.1.7 produces the result in 0.286 second, which means approximately 59% faster. The model is tested using all five instances IV, with both *MiniZinc* 2.1.7 and *MiniZinc* 2.2.1. All the test results are shown in III. To the instance *me_at_the_zoo*, the backend Gurobi is the best one among the others, since it could give the final score after 56.217 seconds while other backends timed-out. When testing with much bigger instances such as *trending_today*, and *video_worth_spreading*, all backends couldn't produce the final results after 600000 milliseconds. The instance *kittens* is the biggest and toughest instance that defeats all the backends, and ends up with the *ERR*.

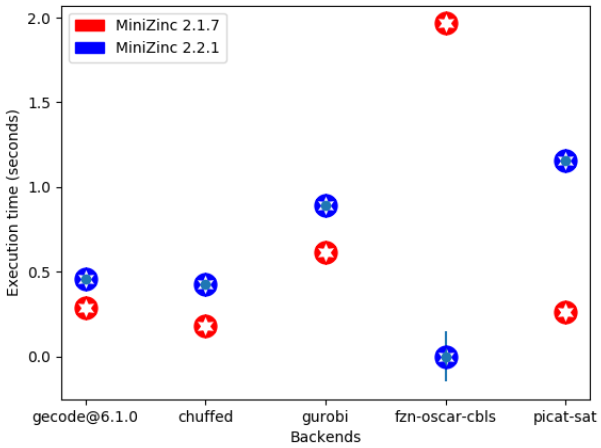

Figure 2: Comparison between *MiniZinc* 2.1.7 and *MiniZinc* 2.2.1 with *warm_up.mzn* instance

a) Experiment with MiniZinc 2.1.7: The model has been tested with *5* instances IV. With the *warm_up* instance, our model gives the total score 562.5, which is better than the given score 462.5 in the Google specification. It's because the caches, which have the minimal latencies, are selected to store the requested videos in the first place. On the *warm_up* instance, we observe that all the chosen *Chuffed* backend wins overall with the execution time is 0.183 second, while the second rank is the *Picat-sat* backend with the execution time is 0.260 second. Other backends such as *Gecode*, *Gurobi*, and *fzn-oscar-cbls* give the results in 0.286, 0.615, and 1.966 seconds, respectively. In the experiment using older *MiniZinc* version, there is *no* time-out backend. In the next step, given a larger instance such as *me_at_the_zoo*, the winner solver is *Gurobi*, with the objective score is 56217. While all other solvers such as *Gecode*, *Chuffed*, *OscaR*, and *Lingeling* don't give any results and time-out. Starting from the medium instances such as *trending_today*, *video_worth_spreading*, and the largest *kittens* instance , backends are time-out and couldn't give any response with the time-out was 600000.

[1] https://github.com/PhucVH888/streamingVideos

Technology	CP		LCG		MIP		CBLS		SAT	
Solver	Gecode		Chuffed		Gurobi		OscaR.cbls		Lingeling	
Backend	Gecode		Chuffed		Gurobi		fzn-oscar-cbls		Picat-sat	
instance	score	time	score	time	score	time	score	time	score	time
warm_up$^{\psi}$	562.5	0.457	562.5	0.424	562.5	0.892	562.5	t/o	562.5	1.154
warm_up*	562.5	0.286	562.5	0.183	562.5	0.615	562.5	1.966	562.5	0.260
me_at_the_zoo$^{\psi}$	–	t/o	–	t/o	607.33	56.217	–	t/o	–	t/o
trending_today* $^{\psi}$	–	t/o	–	t/o	–	t/o	–	t/o	–	t/o
video_worth_spreading* $^{\psi}$	–	t/o	–	t/o	–	t/o	–	t/o	–	t/o
kittens* $^{\psi}$	–	t/o	–	t/o	–	t/o	–	t/o	–	t/o

Table III: Results for our Streaming Videos model. $(^{*})$: *MiniZinc* 2.1.7, $(^{\psi})$: *MiniZinc* 2.2.1

b) Experiment with MiniZinc 2.2.1: In the latest *MiniZinc* version, the winner backend is *Chuffed*, 0.424 second which is similarly to the older version of *MiniZinc*. The backend *Gecode*, which gives the result in 0.457 second, is the second rank. While it takes 0.892 second for *Gurobi* to produce the final score. The worst one is *Picat-Sat*, which gives the result after 1.154 seconds. The remarkable difference between the older and the latest *MiniZinc* version is that in the latest version, *fzn-oscar-cbls* is time-out while the result is given in the older version under the same time-out setting. Similar to the older version, when starting from the medium and the large instances, all the backends are time-out and couldn't give any results before timing-out.

Name	Videos	Cache Servers	Endpoints	Distinct Requests
warm_up	5	3	2	4
me_at_the_zoo	100	10	10	81
video_worth_spreading	10000	100	100	40317
trending_today	10000	100	100	95180
kittens	10000	500	1000	197987

Table IV: Instances of Streaming Videos model.

V. Related work

MiniZinc is a standard modelling language for constraint programming (CP) problems. The motto is *model once, solve anywhere*. Although MiniZinc may contain annotations to communicate with the underlying solver, its model is solver-independent. Most common global constraints, which defined over an arbitrary variables [8], and the separation between model and data are supported. It means a MiniZinc model can be instantiated by different data by defining as a generic template. *MiniZinc* supports sets, arrays, and user-defined predicates, overloading, and some automatic coercions. However, in order to easily map onto many solvers such as G12fd, lazyfd, and Chuffed, *MiniZinc* is still low-level enough. *MiniZinc* models are translated to *FlatZinc*, a low-level solver input language that is the target language for *MiniZinc*. When requiring by a CP solver, *FlatZinc* is translated easily into the required form. Since 2008, *MiniZinc* Challenge has been run every year to compare different solvers on the same benchmarks and to collect as well as develop new *MiniZinc* benchmarks.

Streaming videos is one of the problems at the qualification round in the *Hash Code* 2017 competition running by Google. The *Hash Code* is a coding constests sponsored by Google LLC. The competition is for the programmers who are living in Europe, the Middle East, and Africa. Participants must compete in a group of two to four members as a team [9]. The contest consists of two rounds: the online qualification round and the final round. The *Streaming Video* data consists of four data sets which are in plain text files.

VI. Conclusion and Future Work

In this project, the disadvantage of those backends is the division computation such as $/$ and div, which can be avoided by putting the division computation in the output phase. The real question here is how can the MiniZinc model be improved to instantiate and give the result for the biggest data instance, *kittens*, whose size is up to 5,4 MB in text format. The *Streaming video* problem could be modelled by other modelling language and benchmarked with the same data instances to compare the performance and the efficiency with MiniZinc model.

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