Fuzzy Fast Terminal Sliding Mode Control of Mobile Robot Trajectory Tracking

Ali Mallem and Walid Benaziza

October 22, 2021

Fuzzy Fast Terminal Sliding Mode Control of Mobile Robot Trajectory Tracking

Ali Mallem*,
Walid Benaziza**

** Department of Industrial Engineering, Faculty of Technology University of khenchela, Algeria (email: ali_mallem@hotmail.fr)*
***Electronics Department, Batna 2 University, rue Chahid Boukhlouf Mohamed El Hadi, Batna 05000, Algeria (email: Walid198-ele@hotmail.fr)*

Abstract: In this paper a fast terminal sliding mode control for wheeled mobile robots using a fuzzy logic controller is presented. The proposed strategy combines terminal sliding mode and fuzzy logic approaches. The dynamic tracking errors of the mobile robot are divided in two subsystems, a first order subsystem for the angle error and a second order subsystem for the position error. For the first subsystem, a fast terminal sliding mode control law of the angular velocity is designed in order to stabilize the angle error and assure the finite time. For the second subsystem, a combination of global fast terminal sliding mode control and a fuzzy logic controller is designed in order to stabilize the position error and to optimize the reaching time of the control law. The introduced fuzzy logic controller is used to find a compromise between the reaching time of the sliding surface and the convergence time of the position errors. The designed controller is dynamically simulated by using matlab/simulink. It is shown from the simulation works that the proposed controller offers a fast response and a good trajectories tracking capability for mobile robot.

Keywords: Mobile robot, fast terminal sliding mode, trajectory tracking, fuzzy logic, Lyapunov stability.

1. INTRODUCTION

Many authors have proposed solutions to resolve the problems of wheeled mobile robot tracking control [1,2]. The wheeled mobile robot is a typical nonholonomic system. The stabilization of this kind of robot with restricted mobility to an equilibrium state is in general quite difficult [3]. Such a system suffers nonlinearity and uncertainty problems, so it cannot be stabilized through a fixed feedback. It implies that methods for linear control theory cannot be applied to problems of controlling nonholonomic systems. In recent years, different control techniques have been proposed to control and steer mobile robot on real time trajectory from an initial position to a target. Due to the intrinsic nonlinearity in the mobile robot dynamics and the nonholonomic constraints, nonlinear architectures as adaptive and intelligent methods [4,5], backstepping [6,7], feedback linearization [8] and sliding mode control [9] have been studied.

Until now many methods have been used for trajectory tracking of mobile robots. PID control [10] becomes unstable easily when it is affected by the sensor sensitivity. Fuzzy logic control suffers from the slow response time due to the heavy computation [11]. Feedback linearization approach is limited by the convergence conditions while Lyapunov oriented control [12] is difficult to construct a Lyapunov candidate function [13]. Compared to the approaches described previously, sliding mode control proposes many advantages for mobile robot trajectory tracking such as fast response and robustness in presence of system uncertainties and disturbances [14]. Therefore, it has been used extensively and directed towards the higher order [15] and to multiple surfaces.

To solve the tracking problems, some extensive research have been deployed. Sliding mode control methods were proposed for mobile robots [16], similar problem with bounded disturbances was considered in[9].The control law wish assure the convergence in finite time was introduced in [17].By combining cascaded design and backstepping approach, a tracking controller was designed in [18], the consideration of input torque saturation and external disturbances were introduced.

In new researches of tracking control, a finite time tracking control was deployed in last year’s [19] A global finite-time tracking controller was given for the nonholonomic systems in [19]. The finite-time tracking control is presented in [20].

In order to implement sliding mode control on trajectory tracking of mobile robot, a kinematics model of the mobile robot must be established. The representation of the kinematic equation of mobile robot for the trajectory control can be set up in the Cartesian coordinates. Two sliding surfaces in cartesian coordinates are chosen as for tracking errors in position and heading direction, respectively.

The purpose of this paper is to propose a new sliding mode control to optimize the reaching time and stabilize the position and angle errors.

The paper is organized as follows. Section II presents the kinematics model and the position and posture error model of the mobile robot. The terminal sliding mode controller is designed in section III. The fuzzy cascade sliding mode tracking controller is designed in Section IV. The simulation and analysis of the improved algorithm are presented in Section V. Finally, conclusions are drawn in Section VI.

2. KINEMATICS MODELING OF MOBILE ROBOT

Suppose there is a mobile robot which is positioned on a 2D reference mark in which a global cartesian coordinate system is defined. A local coordinate system fixed on the robot, where its X-axis coincides with its front orientation, is assumed (figure 1). The robot configuration represents the position (x, y) and the orientation θ of the local coordinate system in the global frame (1).

$$p = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \tag{1}$$

The orientation θ is taken counterclockwise from the global X-axis. This configuration stands for the three degrees of freedom which the robot possesses in its world. Let p denote an identity configuration $(0\ 0\ 0)^{T}$..

The robot's kinematics is defined by:

$$\dot{p} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{bmatrix} cos\theta & 0 \\ sin\theta & 0 \\ 0 & 1 \end{bmatrix} q \tag{2}$$

where q represents the control vector $(v,\omega)^{T}$.

Error configuration:

In trajectory tracking of mobile robots, the posture p_r and the velocity q_r are considered and they are written as:

$$q_{\mathrm{r}} = (v_r, \omega_r)^{\mathrm{T}} \tag{3}$$

$$p_r = (x_r, y_r, \theta_r)^{T} \tag{4}$$

If the mobile robot has a posture $p = (x, y, \theta)^{T}$ in a global referential (X,O,Y) and if we consider a referenced posture $p_r = (x_r, y_r, \theta_r)^{T}$, as shown in Fig. 1., then the posture error is given by $p_e = (x_e, y_e, \theta_e)^{T}$.

According to coordinate transformation, the equation of posture error of the mobile robot is described as:

$$p_e = \begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix} \tag{5}$$

For the three-wheeled mobile robot, the trajectory tracking error given in equation (5) can be written as:

$$\begin{cases} x_e = cos\theta(x_r - x) + sin\theta(y_r - y) \\ y_e = -sin\theta(x_r - x) + cos\theta(y_r - y) \\ \theta_e = \theta_r - \theta \end{cases} \tag{6}$$

The equation, characterizing the slip-free rolling of a wheel on the ground, representing the nonholonomic constraints is presented below:

$$\dot{x} sin\theta + \dot{y} cos\theta = 0 \tag{7}$$

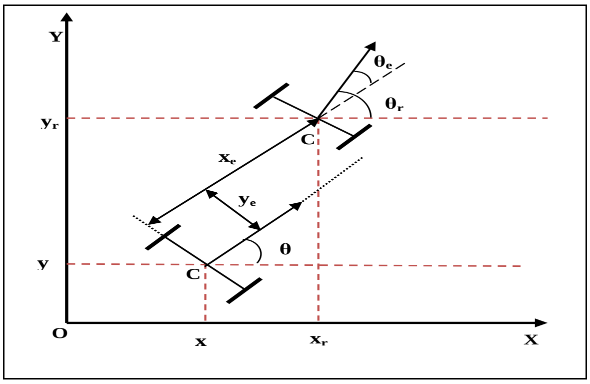

Fig.1. Posture error description

Using the equation (7) and some trigonometry relations, the derivative of the equation (6) can be calculated as below:
(1)

$$\dot{x}_e = \quad y_e\omega - v + v_r cos\theta_e \tag{8}$$

$$\dot{y}_e = -x_e\omega + v_r sin\theta_e \tag{9}$$

$$\dot{\theta}_e = \dot{\theta}_r - \dot{\theta} = \omega_r - \omega \tag{10}$$

According to equations (8), (9) and (10), the following result is obtained:

$$\dot{p_e} = \begin{bmatrix} \dot{x_e} \\ \dot{y_e} \\ \dot{\theta}_e \end{bmatrix} = \begin{bmatrix} y_e\omega - v + v_r cos\theta_e \\ -x_e\omega + v_r sin\theta_e \\ \omega_r - \omega \end{bmatrix} \tag{11}$$

3. DESIGN OF TRACKING CONTROLLER

In this part, two control strategies are proposed, firstly, a fast terminal sliding mode control can make the error angle converge to zero in finite time, and a fast terminal sliding mode converges the sliding surface to the manifold. Secondly, a fuzzy terminal sliding mode can make the sliding surface converge to the manifold in a finite time.

3.1 Fast terminal sliding control design

Fast Terminal sliding mode control can make the system states converge to zero in a finite time. Asymptotical convergence of states under the normal sliding mode is overcome. The convergent characteristic of fast terminal sliding mode control is superior to that of the normal sliding mode control. Moreover, there is no switch function in terminal sliding mode control, therefore, the chattering phenomenon is evitable.

A kind of fast terminal sliding surface is proposed [21] as follows**:**

$$s = \dot{x} + \alpha x + \beta x^{q/p} = 0 \tag{12}$$

The reaching time is giving by the following equation

$$t_s = \frac{p}{\alpha(p-q)} ln \frac{\alpha x(0)^{\frac{p-q}{p}} + \beta}{\beta} \tag{13}$$

When the state x is far away from the origin, then, the convergent time is decided by the fast terminal attractor $\dot{x} = -\beta x^{q/p}$; when the state x approaches the origin, then the convergent time is decided by the equation $\dot{x} = -\alpha x$.

Exponentially, x converges to zero. Therefore, the terminal attractor is introduced in the sliding surface equation (12) and makes the state converge to zero in a finite time. Moreover, the speed of the linear sliding surface is guaranteed. Accordingly, the state can converge to equilibrium speedily and precisely.

3.2 Global fast terminal sliding mode control design

The global fast sliding surface is selected as:

$$s = \dot{s}_0 + \alpha s_0 + \beta {s_0}^{q/p} \tag{14}$$

Where α , $\beta > 0$ and q, p ($q<p$) are positives odd numbers.

3.3 Design of angular velocity control law

According to the equation (11), we have:

$$\dot{\theta}_e = w_r - w \tag{15}$$

we can choose the sliding surface as:

$$s_1 = \theta_e \tag{16}$$

Then we can have:

$$\dot{\theta_e} = -\theta_e - \alpha_1 \theta_e - \beta_1 {\theta_e}^{q/p} \tag{17}$$

Control law can be obtaining as:

$$\omega = \omega_r + \alpha_1 \theta_e + \beta_1 {\theta_e}^{\frac{q}{p}} \tag{18}$$

Theorem: consider the equation (14), there exists the control law (17) such that the system (14) is stable.

Proof: According to equation (16), we choose lyapunov function described by:

$$V = \frac{1}{2}\theta_e^2 \tag{19}$$

The derivate lyapunov can obtain as:

$$\dot{V} = \theta_e \dot{\theta_e} = \left(-\theta_e - \alpha_1 \theta_e - \beta_1 {\theta_e}^{\frac{q}{p}}\right)\theta_e$$
$$= -\theta_e^2 - \alpha_1 \theta_e^2 - \beta_1 {\theta_e}^{\frac{q+p}{p}} \tag{20}$$

Such that p>0 and q>0 are odd integers, α >0 and β >0, we obtain: $\dot{V} \leq 0$, the system is stable.

3.4 Design of the forward velocity control law

Notice that when θe converges to *0*, then $\omega_r = \omega$. Another two state control design are considered in this time:

$$\dot{x_e} = \omega_r y_e - v + v_r \tag{21}$$

$$\dot{y_e} = -\omega_r x_e \tag{22}$$

Switching function can be designed as follows:

$$s_2 = x_e - y_e \tag{23}$$

By designing sliding mode control law that $s \to 0$, that realization x_e converges to y_e in order to achieve $x_e \to 0$ and $y_e \to 0$.
Using the terminal sliding mode control , we can obtain:

$$\dot{s}_2 = -s_2 - \alpha_2 s_2 - \beta_2 {s_2}^{q/p} \tag{24}$$

Using the equations (20), (21):

$$\omega_r y_e - v + v_r + \omega_r x_e = -s_2 - \alpha_2 s_2 - \beta_2 {s_2}^{q/p} \tag{25}$$

Then, the forward velocity control law can be obtained as:

$$v = v_r + \omega_r x_e + \omega_r y_e - s_2 - \alpha_2 s_2 - \beta_2 {s_2}^{q/p} \tag{26}$$

The Lyapunov candidate function can be defined as:

$$v_S = \frac{1}{2}{s_2}^2 \tag{27}$$

By differentiating this equation:

$$\dot{v_S} = s_2\, \dot{s}_2 = s_2(-s_2 - \alpha_2 s_2 - \beta_2 {s_2}^{q/p})$$
$$= -{s_2}^2 - \alpha_2 {s_2}^2 - \beta_2 {s_2}^{(\frac{q}{p}+1)} \tag{28}$$

As p and q are positives odd and $p > q$ then:

$$-{s_2}^2 - \alpha_2 {s_2}^2 - \beta_2 {s_2}^{(\frac{q}{p}+1)} \leq 0 \tag{29}$$

4. FUZZY SLIDING MODE CONTROL

In this part, a fuzzy controller is implemented for keeping an accurate finite time control. In the precedent part, the control law of the forward velocity assures the convergence of the sliding surface to zero, but don't assure the convergence of the errors x_e and y_e to zero. the reaching time depend of the parameter α , when we increase the value of the parameter α , we obtain a convergence in very little time but with considerable errors (x_e and y_e). When we lower the value of the parameter α , we obtain a considerable reaching time with low errors. Therefore for assure a very low reaching time with low errors, a fuzzy controller is implemented to assure this conditions.

4.1 Control strategy

This strategy is based on the same control laws of the precedent part. The fuzzy logic controller has one input and one output (the value of the parameter α).

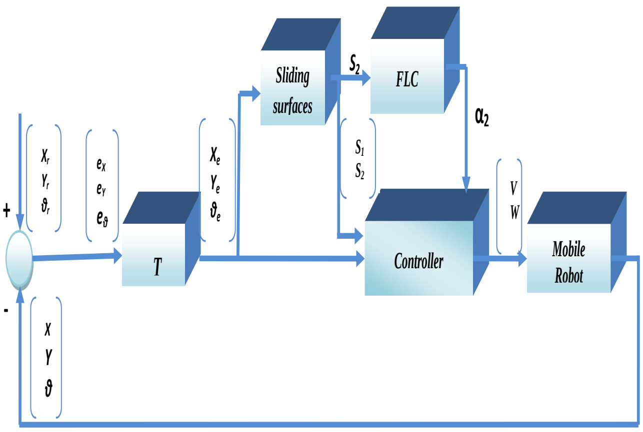

Fig.2. Fuzzy sliding mode control strategy

4.2 Choice of fuzzy units

We choose the forms of membership functions and their distribution on the speech inverse. The input variable membership is partitioned into 3 fuzzy units (N, Z, P), and the output variable membership is partitioned into 3 fuzzy units (S, M, L).

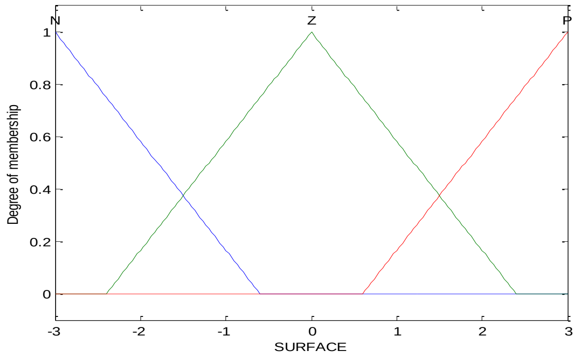

Fig.3. Membership function input (surface S_2)

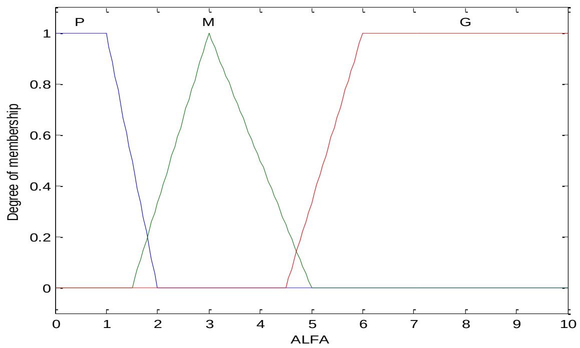

Fig. 4. Membership function output (parameter α_2)

4.3 Choice of inference rules

The following rules represent the inference rules used in this work. These rules are obtained while being based on the system knowledge to control. The table significance indicating the linguistics values names are:

N: Negative, Z: Positive, Z: Zero.
S: Small, M: Medium, L: Large.

IF Surface is N Then Alfa is L

IF Surface is Z Then Alfa is S

IF Surface is P Then Alfa is M

5. SIMULATION RESULTS

According to the control law established in Section 3 and 4, the simulation using matlab is applied on the mobile robot system. The plant to be controlled is the differential between the desired and real trajectories of the mobile robot. The simulations are subdivided in two parts: firstly using the fast terminal sliding mode and secondly the fuzzy cascade fast terminal sliding mode.

5.1 Cascade fast terminal sliding mode

Let us consider the following values:

$v_r = 1$ m/s, $\omega_r = 1$ rad/s and $r = \frac{v_r}{\omega_r}$

and the reference posture $p_r = (x_r, y_r, \theta_r)^T$ as follows:

$$\begin{cases} x_r = r\,cos(\omega_r t) = \cos(t) \\ y_r = r\,sin(\omega_r t) = \sin(t) \\ \theta_r = \omega_r t = t \end{cases}$$

Let us consider the following values for the various parameters.

Table.1 Parameters values

α_1	4	α_2	0.5
β_1	8	B_2	3
q_1	5	q_2	3
p_1	7	p_2	5

The initial errors are: (-1.8, 0.3,- pi/5)

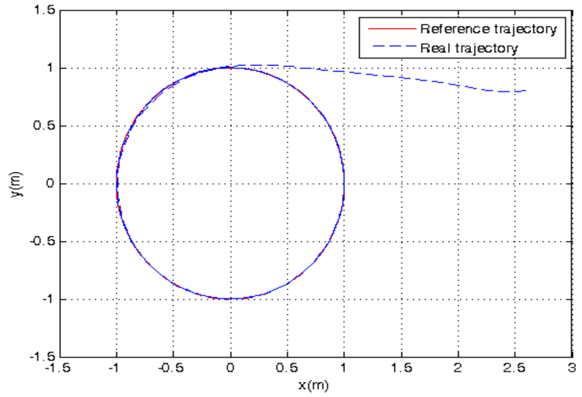

Fig.5. Circular trajectory tracking

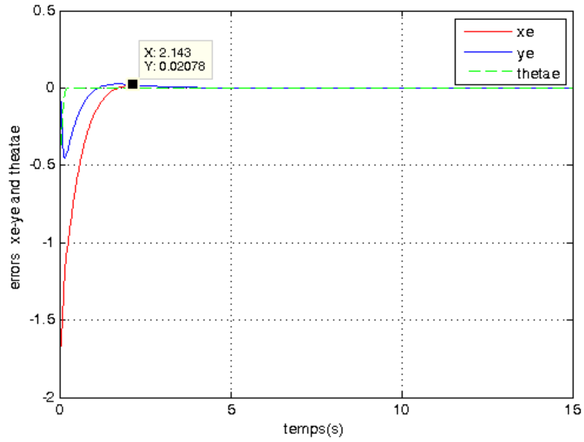

Fig.6. Trajectory tracking errors

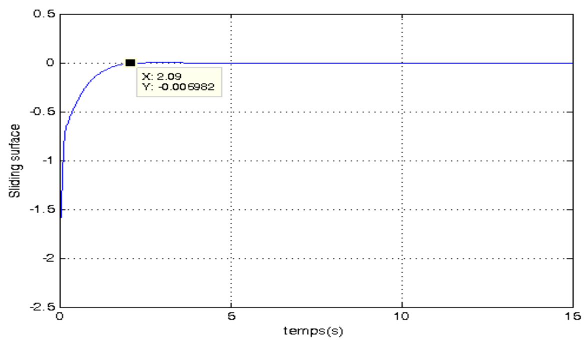

Fig.7.Sliding surface (S_2)

Figure (5) show that the mobile robot track the reference trajectory within a limited time. In figure (6),the error θ_e converges to zero in split second time, but the errors y_e and x_e converge to zero in considerable time (>2s), the figure (6) show also the overtaking of the errors y_e and x_e .Figure (7) presents the second sliding surface (S_2) and presents the reaching time (t_s=2.09 s).

To solve the problem of reaching time of sliding surface and the convergence time of the errors, a fuzzy controller is proposed in next part of simulation.

5.2 Fuzzy Cascade fast terminal sliding mode

The reference posture $p_r = (x_r, y_r, \theta_r)^T$ is as follows:

$$\begin{cases} x_r = r\cos(\omega_r t) = \cos(t) \\ y_r = r\sin(\omega_r t) = \sin(t) \\ \theta_r = \omega_r t = t \end{cases}$$

Table.2 Parameters values

α_1	4	B_2	3
β_1	8	q_2	3
q_1	5	p_2	5
p_1	7		

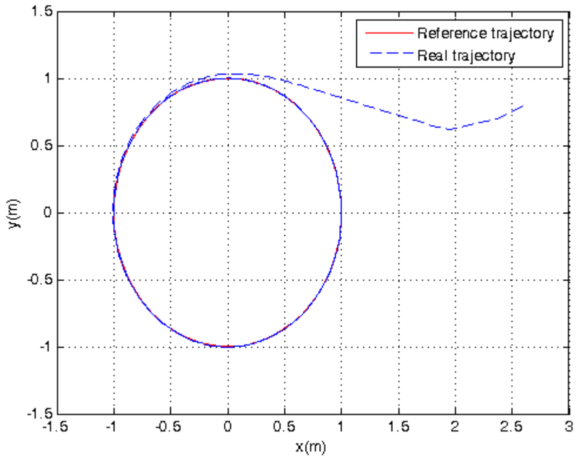

Fig.8. Circular trajectory tracking

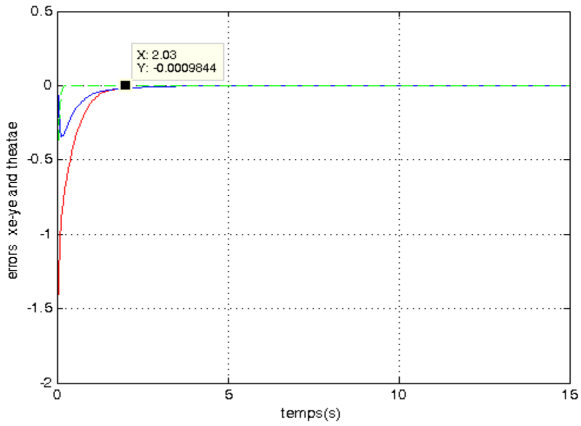

Fig.9. Trajectory tracking errors

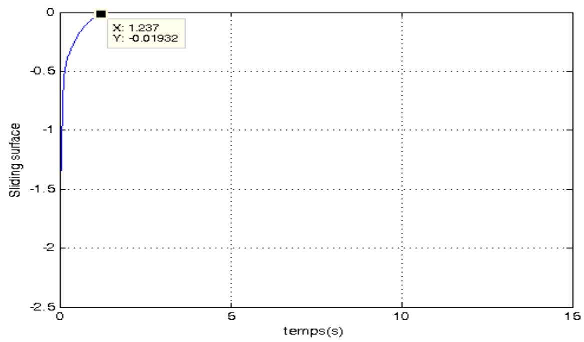

Fig.10. Sliding surface (S_2)

This part of result simulation illustrate the ability and potency of the control approach introduced (fuzzy logic control), when the figure (9) show that the errors x_e and y_e converge to zero in low time (t_c=2s).

Figure (10) illustrate the sliding surface, when the reaching time is very low (t_s=1.2 s), therefore; the fuzzy sliding mode control introduced in this article guarantees the convergence of errors in short time and a very short reaching time .

The simulation results using this control strategy, presented in this article, allow a designed control law significantly improving convergence speed, but also minimal tracking error when using a sliding mode control with a stronger anti-interference ability. It is easy to find, direction angle of convergence which reflects the fast terminal sliding mode control convergence characteristics.

From the results obtained x_{er-tot}, y_{er-tot}, θ_{er-tot}, the mean squared error (MSE) can be calculated as follows:

Table.3 error MSE and comparative study of the results

	α_2=0.5	α_2=2	α_2=10	fuzzy par
MSE	0,0779	0,0604	0.3952	0.0423
Reaching time	2.09 s	1.076 s	0.25 s	1.237 s
Convergence time (t_c)	2.145 s	3.297 s	3.451 s	2.03 s

The table above demonstrates the efficiency of the proposed control, either the first approach control (cascade fast terminal sliding mode), or the second approach (fuzzy cascade sliding mode). For the first approach, the variation of the parameter α_2 showed that is impossible to ensure the three criteria (MSE, reaching time and convergence time) at the same time. Therefore; the fuzzy logic approach can ensure the three criteria, but it’s not an ideal case.

6. CONCLUSION

In this paper a new approach is proposed to ensure the trajectory tracking of mobile robot in finite time and minimize the errors. The fuzzy logic approach prove that can make the system converges to the reference in a short reaching time, and the errors of position converge to zero in a short convergence time, also minimize the MSE error.

According to the simulation results, the proposed sliding mode control is an important method to deal with the system which has uncertainties and nonlinearities. These proposed algorithms demonstrate a good tracking performance. In spite of large initial error, the robot posture converges to the desired trajectory. Simulation results have demonstrated that the control method can not only ensure the high accuracy of the mobile robots trajectories tracking, but also keep high stability and reliability.

REFERENCES

[1] Kanayama, Y., Kimura, Y., Miyazaki, F. and Noguchi, T. “A stable tracking control method for an autonomous mobile robot”. In Proceedings of IEEE Conference on Robotics and Automation, vol. 1, 1990,pp. 384-389.

[2] Ti-Chung, L., Kai-Tai, S., Ching-Hung, L. and Ching-Cheng, T. “Tracking control of mobile robots using saturation feedback controller”. In Proceedings of IEEE International Conference on Robotics and Automation, vol. 4, 1999, pp. 2639-2644.

[3] Fukao, T., Nakagawa, H. and Adachi, N. “Adaptive tracking control of a nonholonomic mobile robot.” IEEE Transactions on Robotics and Automation. 16 (5), 2000, pp 609–615.

[4] Hwang, C. L. “A novel Takagi-Sugeno-based robust adaptive fuzzy sliding-mode controller”. IEEE Transactions on Fuzzy Systems. 12 (5), 2004, pp 676–687.

[5] Mohareri, O., Dhaouadi, R., Shirazi, M.M. “Intelligent neural network based controllers for path tracking of wheeled mobile robots: A comparative analysis”. IEEE International Workshop on Robotic and Sensors Environments, 2010, pp. 1-6.

[6] Nganga-Kouya,D. and Okou,F.A. “Adaptive backstepping control of a wheeled mobile robot”. 17th Mediterranean Conference on Control and Automation. 2009,pp. 85-91.

[7] Ching-Chih, T. and Shiang-Yun, J. “ Trajectory tracking and regulation of a self-balancing two-wheeled robot: A backsteppin sliding-mode control approach”. Proceedings of SICE Annual Conference, 2010,pp. 2411-2418.

[8] Song, K.T., Lee, C.H. and Teng, C.C., “Tracking control of unicycle-modeled mobile robots using a saturation feedback controller”. IEEE Transactions on Control Systems Technology, 2001, pp. 305-318.

[9] Yang, J.M. and Kim, J.H. “Sliding mode control for trajectory tracking of nonholonomic wheeled mobile robots”. IEEE Transactions on Robotics and Automation. 15 (3), 1999,pp 578–587.

[10] Atherton, D.P. and Majhi, S. “Limitations of PID controllers”. In Proceedings of the American Control Conference, vol. 6, 1999, pp. 3843-3847.

[11] Butt, C. and Rahman, M.A. “Limitations of simplified fuzzy logic controller for IPM motor drive”. Industry Applications Conference, vol. 3, 2004, pp. 1891-1898.

[12] Orlov, Y. “Discontinuous Systems: Lyapunov Analysis and Robust Synthesis under Uncertainty Conditions”, Springer-Verlag, London, UK.,2010.

[13] Orlov, Y. “Discontinuous Systems: Lyapunov Analysis and Robust Synthesis” under Uncertainty Conditions, Springer-Verlag, London, UK, (2010).

[14] Defoort, M., Floquet, T., Kokosy, A. and Perruquetti, W.” Integral sliding mode control for trajectory tracking of a unicycle type mobile robot”. Integrated Computer Aided Engineering. 13 (3), 2006, 277-288.

[15] Dinuzzo, F. and Ferrara, A. “Higher order sliding mode controllers with optimal reaching”, IEEE Trans. Autom. Control 54 (9), 2009, 2126–2136.

[16] Chwa. D, “Sliding-mode tracking control of nonholonomic wheeled mobile robots in polar coordinates”, IEEE Trans. Contr. Syst. Technol. 12 (4) (2004) 637–644.

[17] Tian. Y.P, Li. S.H, “Exponential stabilization of nonholonomic dynamic systems by smooth time-varying control”, Automatica 38 (7) (2002) 1139–1146.

[18] Fu. J, Chai. T.Y, Su. C.Y, Jin. Y, “Motion/force tracking control of nonholonomic mechanical systems via combining cascaded design and backstepping”,Automatica 49 (12) (2013) 3682–3686.

[19] Wu. Y.Q, Wang. B, Zong. G.D, “Finite-time tracking controller design for nonholonomic systems with extended chained form”, IEEE Trans. Circ. Syst. 52 (11) (2005) 798–802.

[20] Du. H.B, Li. S.H, Qian. C.J,” Finite-time attitude tracking control of spacecraft with application to attitude synchronization”, IEEE Trans. Autom. Contr. 56 (11) (2011) 2711–2717

[21] Park, KB. and Tsuiji, T. “ Terminal sliding mode control of second-order nonlinear uncertain systems”. International Journal of Robust and Nonlinear Control, 1999, (11), 769 -780.