Mathematics of 21st Century: A Personal View

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Extended Abstract.

Mathematics of **19th century** and before was rich in content, intuition, strong ideas, motivation from natural sciences, and had a coherent view of all fields of mathematics (geometry, algebra, analysis, number theory, ...).

Mathematics of **20th century** added three important aspects: mathematical **logic** as the mathematical meta-theory of mathematics; formalism and abstractness and reduction to first principles (**"Bourbakism"**); and the notion of **computing** (both as an exact mathematical notion and as the foundation of the revolutionary device "programmable computer"). In fact, the three aspects have a lot to do with each other and are fundamentally interrelated. However, in practice, the three aspects were (and still are) pursued by three different communities: The deep insights of logicians on the nature of mathematics are hardly used in ever-day practice of "working mathematicians"; "pure mathematicians" are often proud that they do not touch the computer except for writing e-mails, typing in LaTeX, and using web search; and computer scientists often think that what they are doing does not need mathematics or, even, is opposite to mathematics.

In my view, as indicated by trends in late 20th century, in **21st century** mathematics will evolve as - or return to be - a **unified body** of mathematical logic, abstract structural mathematics, and computer mathematics with no boundaries between the three aspects. "Working mathematicians" will have to master the three aspects equally well and integrate them in their daily work. More specifically, working in mathematics will proceed on the "object level" of developing new mathematical content (abstract knowledge and computational methods) and, at the same time, on the "meta-level" of developing automated reasoning methods for supporting research on the object level. This "massage of the mathematical brain" by jumping back and forth between the object and the meta-level will guide mathematics onto a new level of sophistication.

Symbolic computation is just a way of expressing this general view of mathematics of the 21st century and it also should be clear that **software science** is just another way of expressing the algorithmic aspect of this view.

In the talk, we will exemplify the spirit of this new type of mathematics by a report on the **Theorema** system being developed in the speaker's research group. Theorema is both a logic and a software frame for doing mathematics in the way sketched above. On the object level, Theorema allows to prove and program within the same logical frame and, on the meta-level, it allows to formulate reasoning techniques that help proving and programming on the object level. In particular, we will show how this type of doing mathematics allows to mimic the invention process behind the speaker's theory of Gröbner bases, which provides a general method for dealing with multivariate nonlinear polynomial systems.